ADJOINT BASED A POSTERIORI ERROR ESTIMATES USING DATA COMPRESSION

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Abstract. One of the major computational burdens in the application of adjoint techniques to time-dependent nonlinear problems is the need to store the full forward approximation to define the adjoint of the linearized forward problem and to evaluate the dual-weighted residual [1, 2]. An approach for mitigating the storage cost is checkpointing, whereby the forward solution is stored at a series of carefully selected time nodes. During an adjoint computations these checkpoints are used to reconstruct the forward solution, recomputing on each subinterval as needed. A notable implementation of the checkpointing scheme is the REVOLVE [3] algorithm. This method minimizes the number of recomputations of the forward solution at any timestep allowing for a fixed storage budget and fixed number of time steps. Recent versions of this algorithm permit a varying number of timesteps with bounded maximum number of recomputations, albeit with a slowly growing storage cost [4]. These approaches successfully reduce the storage cost when the exact solution to the forward problem is required. However, the cost of recomputing the forward solution may be to great of a burden for some applications.

We relax the assumption that the computed forward solution is needed to evaluate the adjoint when used in goal-oriented error estimation. We show that the accuracy of the forward solution has limited effect on the accuracy of the error estimate, and use this to develop a method for approximation of the forward solution that gives reasonably good error estimates. To this end, a number of data compression algorithms are proposed where the storage cost of the approximate foward solution is small compared to storing the exact forward solution. Yet the accuracy of the resulting error estimates are of good quality and no recomputations of the forward solution are required.

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