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HIERARCHICAL MOMENT CLOSURE APPROXIMATION OF THE BOLTZMANN EQUATION

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Abstract. The Boltzmann equation (BE) is the classical model in the kinetic theory of (mono-atomic) fluids, describing rarefied flow by modeling deviations of the velocity distribution from a local equilibrium, thus, accounting for the transitional molecular/continuum regime. The BE provides an evolution equation for the one-particle marginal, viz., the probability density of particles in phase (position/velocity) space. The BE has several fundamental structural properties, notably, conservation of mass, momentum and energy, Galilean invariance and decay of an entropy functional (the celebrated H-theorem). These structural properties underly the connection between the BE and conventional continuum models: all conventional continuum models, such as the Navier-Stokes-Fourier system [1], can be derived as limits of the BE.

The BE poses a formidable challenge for numerical approximation methods, on account of its high dimensional phase-space setting: for a problem in N spatial dimensions, the single molecule phase-space is 2N dimensional. Away from the fluid dynamical regime numerical approximations of kinetic systems are predominantly based on particle methods, such as the Direct Simulation Monte Carlo (DSMC) method. However, the phase-space description of the system results in the prohibitive computational cost of DSMC in the fluid dynamical limit. Moreover, from an approximation perspective, DSMC can be inefficient since it is inherent to the Monte-Carlo process that the approximation error decays only as $n^{-1/2}$ for the number of simulation molecules n [2]. Hence, efficiently modeling gases in the transition regime between the free molecular flow and fluid dynamics remains difficult.

An alternative strategy to describe deviations from fluid dynamics is by means of moment-closure approximations [3, 4]. In moment-closure approximations, the BE is projected onto a polynomial space, in the velocity dependence, and the system is closed by providing an approximation to the one-particle marginal based on the same polynomial space. This procedure can in fact be conceived of as a Galerkin approximation. The closure is chosen such that the structural properties of the BE are retained. From an adaptive approximation standpoint, the resulting hierarchical structure of the the moment closure system presents promising potential for rigorous model adaptivity. However, fundamental challenges remain to be addressed.

This work applies the moment method onto the BE to derive a closed hierarchy of moment systems that retain structural features of the system in question and address the aforementioned issues. In addition opportunities pertaining to goal-oriented adaptive modeling provided by the hierarchical structure exhibited by the resulting closed systems of moment equations will be discussed.

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