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ONLINE GREEDY REDUCED BASIS CONSTRUCTION USING DICTIONARIES

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Abstract. As numerical simulations find more and more use in real-world scenarios and industrial applications, demands concerning efficiency and reliability increase as well. Especially scenarios that call for real-time simulations or multi-query evaluations of partial differential equations (PDEs) often require means of model order reduction. Examples for such scenarios are optimal control and optimization settings.

The Reduced Basis (RB) method [7] provides model order reduction for a special class of PDEs, so-called parameterized partial differential equations (PPDEs), (in the weak and discretized form) given as

$$B_h(u_h(\boldsymbol{\mu}), v_h; \boldsymbol{\mu}) = L_h(v_h; \boldsymbol{\mu}) \quad \forall v_h \in X_h, \tag{1}$$

for $u_h(\boldsymbol{\mu}) \in X_h$, a parameter vector $\boldsymbol{\mu} \in \mathcal{P} \subset \mathbb{R}^p$ and a suitable given discrete function space X_h. Here, $B_h : X_h \times X_h \times \mathcal{P} \to \mathbb{R}$ denotes a given parameterized bilinear form and $L_h : X_h \times \mathcal{P} \to \mathbb{R}$ denotes a given parameterized linear form. The RB method provides model order reduction for this class of PDEs by splitting all computations in two parts: The first part is a possibly time-consuming so-called *offline phase* that comprises all computations involving the grid width of the discretization. The second part is a usually very rapid *online phase* that performs computations only on a reduced system.

While the RB method has been applied successfully to elliptic [7], parabolic [5] and hyperbolic [4] equations and systems, efforts had to be made to apply them to problems with strong sensitivity with respect to the parameter $\boldsymbol{\mu}$. These efforts comprise the hp-method [3] where the parameter space $\mathcal{P}$ is adaptively split and multiple reduced bases are built. While this approach is very efficient during the online phase, it suffers from very high demands concerning runtime and storage during the offline phase.

In our contribution, we will introduce a new approach to the aforementioned problem that holds some similarity with the local greedy method introduced in [6]. Our method

constructs a huge "dictionary" of potential basis vectors during the offline phase. During the online phase, a small, parameter-dependent basis is constructed using a Greedy technique. By exploiting a posteriori error estimation for the basis construction during the online phase, we obtain optimal bases regarding basis size. We present different possible algorithms both for offline dictionary and online basis construction and present theoretical and numerical complexity studies. Furthermore, we present numerical investigations of our approach regarding final basis size, approximation quality and online runtime and a comparison to standard RB methods.

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