

PROBABILITY AND VARIANCE-BASED STOCHASTIC DESIGN OPTIMIZATION OF A RADIAL COMPRESSOR CONCERNING FLUID-STRUCTURE INTERACTION

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Abstract. Within the design development phases the Design for Six Sigma concept optimizes a design such that the products conform to Six Sigma quality. Which means that robustness and reliability are explicit optimization goals even with variations e.g. in manufacturing, design configuration and environment.

Since the engineering of radial compressors the improvement of specific physical behavior, especially the efficiency, is one of the key issues. In conventional engineering the design is improved by evaluating design response and making design changes based on experience, intuition or guess. Due to the introduction of virtual prototyping the radial compressor analysis have a very high degree of complexity and desired improvements are hard to reach with conventional trial and error procedure.

In the reliability-based robust design optimization, the optimization problem

$$\begin{aligned}
& f(d_1, d_2, \dots, d_{n_d}) \rightarrow \min \\
& g_k(d_1, d_2, \dots, d_{n_d}) = 0; \quad k = 1, m_e \\
& h_l(d_1, d_2, \dots, d_{n_d}) \geq 0; \quad l = 1, m_u \\
& 1 - \frac{P(\mathcal{F})}{P^t(\mathcal{F})} \geq 0; \quad P(\mathcal{F}) = \int_{g_j(\mathbf{x}) \leq 0}^{n_r} f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} \\
& \frac{\sigma_{L_j}}{\sigma_L^t} - 1 \geq 0; \quad \sigma_{L_j} = \frac{g_j(X_i) - \bar{X}_j}{\sigma_{X_j}}; \quad j = 1, m_g \\
& d_i \in [d_l, d_u] \subset \mathbb{R}^{n_d}; \quad d_{l_i} \leq d_i \leq d_{u_i}; \quad d_i = E[X_i]
\end{aligned} \tag{1}$$

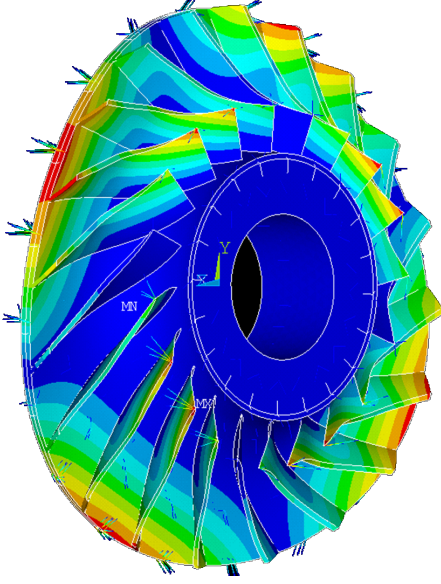


Figure 1: Eigenvalue analysis with ANSYS Workbench to calculate the state condition $g(\mathbf{X})$ for the reliability analysis.

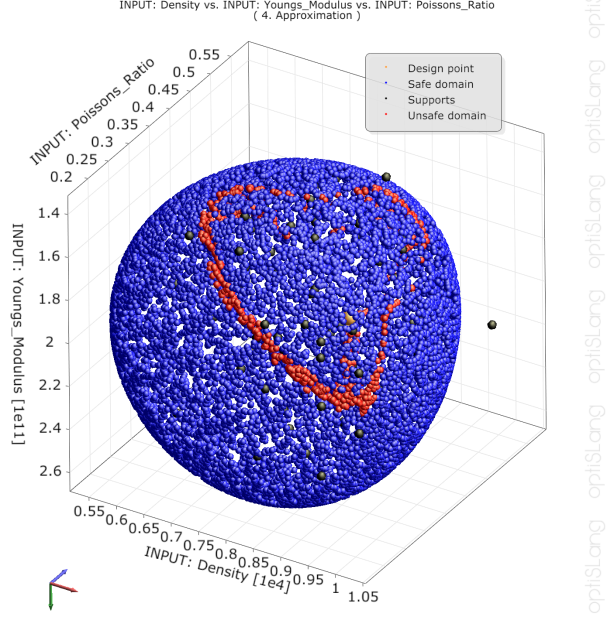


Figure 2: The oscillatory phenomenon of the first eigen frequencies with the rotational velocity of the rotor results in a small non-closed failure domain in the space of the random material parameters.

can be enhanced by additional stochastic restrictions regarding the sigma levels σ_{L_j} ensuring that the probability of failure can not exceed the target probability, for example $P(\mathcal{F}) \leq 3.4 \cdot 10^{-6} = P^t(\mathcal{F})$. Problem (1) is solved as a combination of a deterministic optimization in the n_d -dimensional design space, whereby the design parameters $\mathbf{d} = E[\mathbf{X}]$ are the means of the n_r random influences \mathbf{X} for every deterministic design with the joint probability density function of the basic random variables $f_{\mathbf{X}}(\mathbf{x})$ and m_g limit state functions $g_j(\mathbf{x}) \leq 0$.

This procedure leads in general to an inefficient double loop with a large number of design evaluations. The most general way for reducing the required number of design evaluations is the application of an iterative decoupled loop approach (see e.g. Chen et al., 2003). This effective approach can be enhancement by updating the constrains during the internal optimization using statistical moments in place of the exceedance probability. Essentially, by means of this transformation, the probability-based highly nonlinear and non-differentiable constrains may be more well conditioned for the optimization approach. In this paper an efficient iterative decoupled loop approach is provided for reducing the necessary number of design evaluations.

For an efficient reliability assessment, a new multi-domain adaptive design of experiment in combination with importance sampling and directional sampling is introduced to improve the accuracy and predictability of surrogate models, commonly used in applica-

tions with several limit state conditions (see Roos, 2011). Furthermore, the identification of the failure domains using the directional sampling procedure, the pre-estimation and the priori knowledge of the probability level is no longer required. Therefore the presented method is particularly suitable to solve reliability-based structural design optimization problems considering uncertainties with ever-changing failure probabilities of the nominal designs.

The applicability for real case applications is demonstrated exemplary for a radial compressor, used in power plants or aircraft engines. In the presented example the target of the optimization process is to maximize the efficiency of the turbine engine with respect to a limitation of the maximal v. Mises stress. Additional constraints are defined by resonance of any eigen frequency with the rotational velocity of the rotor (see Figures 1 and 2).

References

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