ERROR CONTROL FOR NONCONFORMING FINITE ELEMENT METHODS FOR THE AFFINE OBSTACLE PROBLEM

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Abstract. In a simple obstacle problems with affine obstacle and the weak regularity assumption of Lebesgue integrable Lagrange multiplier in $L^2(\Omega)$, the Crouzeix-Raviart nonconforming finite element methods allows for some best-approximation result for the gradients plus terms which explicitly converge linearly as the maximal mesh-size approaches zero. For instance in the case of singular solutions when the later contributions are less dominant, this allows for a comparison result that states that the energy norm of the nonconforming finite element solution is essentially smaller than that of the conforming one. The proofs are based on particular conforming companions plus standard arguments from convex analysis and variational inequalities such as the complementary conditions.

The a posteriori error control follows the general approach of D.Braess and designs a new discrete Lagrange multipliers which allow the computation of a guaranteed upper error bound. The practical performance in numerical examples and the question of efficiency concludes the paper.