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J. P. Moitinho de Almeida, P. Díez, C. Tiago and N. Parés (Eds)

ROBUST ERROR CONTROL FOR PHASE FIELD MODELS PAST TOPOLOGICAL CHANGES

SÖREN BARTELS

Department of Applied Mathematics,
University of Freiburg, Germany

Abstract. Phase field models are often used to describe the evolution of submanifolds, e.g., the Allen-Cahn equation approximates motion by mean curvature and more sophisticated phase field models provide regularizations of the Willmore flow and other geometric evolution problems. The models involve small regularization parameters and we discuss the dependence of a priori and a posteriori error estimates for the numerical solution of the regularized problems on this parameter. In particular, we address the question whether robust error estimation is possible past topological changes. We provide an affirmative answer for a priori error estimates assuming a logarithmic scaling law of the time averaged principal eigenvalue of the linearized Allen-Cahn or Ginzburg-Landau operator about the exact solution. This scaling law is confirmed by numerical experiments for generic topological changes. The averaged eigenvalue about the approximate solution enters a posteriori error estimates exponentially and therefore, critical scenarios are detected automatically by related adaptive finite element methods. The devised scheme extracts information about the stability of the evolution from the approximate solution and thereby allows for a rigorous a posteriori error analysis.