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GOAL-ORIENTED ERROR ESTIMATION FOR NONLINEAR PARABOLIC EQUATIONS BASED ON THE RECONSTRUCTION OF EQUILIBRATED FLUXES

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Abstract. We derive a goal-oriented estimate of the error in finite element approximations of nonlinear parabolic equations based on a representation that involves an equilibrated flux reconstruction in space variable of the solution to the primal and to the (linear) dual backward-in-time problem. The error estimate can be applied to any arbitrary finite element discretization of the primal problem that admits a flux reconstruction in space variable satisfying some local space-time conservation and approximation properties [1]. We assume that an implicit-in-time Euler-type scheme is employed for the primal problem. The adjoint problem is then approximated using the same type of discretization scheme in time and a discontinuous Galerkin (dG) finite element method in space on the same mesh as the one used for the primal problem. Owing to the local conservation property of the dG, reconstructed equilibrated fluxes associated with the dual problem [2] can be straightforwardly obtained for the calculation of the error estimates. In fact, the dG method naturally produces fully computable elementwise contributions to the error, which are accurate even on the original mesh since the support of the basis functions coincides with the elements, see [3]. In this talk, we prove, and confirm with numerical experiments, that the proposed error estimator is asymptotically exact.

REFERENCES

[1] V. Dolejší, A. Ern, and M. Vohralík. A framework for robust a posteriori error control in unsteady nonlinear advection-diffusion problems. Technical report, Sept. 2012. HAL: hal-00652979, version 2.

[2] A. Ern, S. Nicaise, and M. Vohralík. An accurate $\mathbf{H}(\mathrm{div})$ flux reconstruction for discontinuous Galerkin approximations of elliptic problems. *C. R. Math. Acad. Sci. Paris*, 345(12):709–712, 2007.

[3] I. Mozolevski and S. Prudhomme. Goal oriented estimates for elliptic equations based on equilibrated fluxes reconstruction. In preparation.