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ADAPTIVE TIME-STEPPING FOR CAHN-HILLIARD-TYPE EQUATIONS WITH APPLICATION TO DIFFUSE-INTERFACE TUMOR-GROWTH MODELS

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Abstract. Many tumor-growth phenomena can be considered as multiphase problems. Employing the continuum theory of mixtures, phase-field tumor-growth models can be derived with diffuse interfaces. The chosen form of the Helmholtz free-energy leads to equations of the Cahn-Hilliard type. Such nonlinear fourth-order partial-differential equations are time-dependent, and their solutions exhibit alternating fast and slow variations in time. It is therefore of prime importance to use adaptive time-stepping to efficiently simulate the entire dynamics of the system [1].
In this contribution, we consider a thermodynamically consistent four-species model of tumor growth in which the energy is non-increasing and total mass is conserved [2]. In order to inherit the two main characteristics of the system at the discrete level, we propose a gradient-stable time-stepping scheme with second-order accuracy. Mixed finite elements are used for spatial discretization. For this discretization, we discuss simple adaptive time-stepping strategies as well as a posteriori error estimates in time. Furthermore, we present illustrative numerical results.

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