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NUMERICAL INTEGRATION OF WEAK FORM IN EMBEDDED INTERFACE METHODS

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Abstract. The numerical integration of weak form over the elements that are crossed by discontinuities in embedded interface methods is addressed in this work. Since these methods lead to complex shaped cut volumes, integration of weak form requires an efficient method for integration of polynomials over arbitrary polyhedra. Most widely, volume decomposition [1] or moment fitting methods [2] are used for such integrations. In this work, we present an efficient and robust method, based on the divergence theorem, for integration of polynomials over polyhedra. For a scalar function $\mathcal{F}$, using the divergence theorem, the integration over $\mathcal{R} \subset \mathbb{R}^3$ whose boundary is given by $\mathcal{S}$ can be written as,

$$\int_{\mathcal{R}} \mathcal{F} dV = \int_{\mathcal{S}} \mathcal{G} n_x \, dA; \quad where \quad \mathcal{G} = \int_{\kappa}^{\mathcal{S}} \mathcal{F} dx \tag{1}$$

where κ is an arbitrary reference point. $\mathcal{G}$ is evaluated by integrating $\mathcal{F}$ using one-dimensional Gauss quadrature, and then to compute the required integral, $\mathcal{G}$ is integrated using another set of Gauss quadratures defined on surfaces of the polyhedra. The method is extremely easy to implement, and we show through numerical examples that it is efficient as well.

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