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J. P. Moitinho de Almeida, P. Díez, C. Tiago and N. Parés (Eds)

QUANTIFYING THE COMPUTABILITY OF THE LORENZ SYSTEM USING A POSTERIORI ANALYSIS

BENJAMIN KEHLET AND ANDERS LOGG

Center for biomedical computing, Simula Research Laboratory
and Department of Informatics, University of Oslo
e-mail: benjamik@simula.no, logg@simula.no

Abstract. It is well known that the computation of accurate trajectories of the Lorenz system is a difficult problem. Computed solutions are very sensitive to the discretization error determined by the time step size and polynomial order of the method, as well as round-off errors.

In this work, we show how round-off errors limit the computability of the Lorenz system and quantify exactly the length of intervals over which solutions can be computed, expressed in terms of the floating point precision. Using adjoint-based a posteriori error analysis techniques, we estimate the stability of computations with respect to initial data, discretization, and round-off errors, respectively.

The analysis is verified by computing accurate solution on the time interval [0, 1000] using a very high order (order 200) finite element method and very high floating point precision (400 digits).