p-adaption for compressible flows

D. Ekelschot, J Peiró, S.J. Sherwin, D. Moxey and C. Biotto

Department of Aeronautics
Imperial College London
South Kensington Campus
London SW7 2AZ

Abstract

We present a p-adaptive method which takes advantage of the ability of a discontinuity sensor used to quantify the difference between the actual solution (p) and the projected reduced one ($p-1$) in order to vary the polynomial resolution in an element. The value of the sensor in an element is defined as:

$$S_e = \frac{||\rho_e^p - \rho_e^{p-1}||_{L_2}}{||\rho_e^p||_{L_2}}$$

where ρ_e^p and ρ_e^{p-1} are the average solutions of degree p and $p-1$ respectively on the same element. The polynomial degree is decreased when a discontinuity is present in order to avoid oscillations and increased when a high gradient is identified to improve the accuracy. This procedure allows the simulation to adapt to the flowfield, increasing the accuracy of the solution only where needed and, as a consequence, reducing the computational cost required for solving the problem.

Initially, a converged linear solution is obtained after which the sensor in each element is calculated. Based on the determined sensor value and the pre-defined sensor thresholds, the degree of the polynomial approximation in each element is increased, reduced or maintained and a new converged solution is obtained. The sensor distribution is divided into four zones:

$$p_e = \begin{cases} p_e - 1 & \text{if} \;\; s_e > s_{ds} \\ p_e + 1 & \text{if} \;\; s_{sm} < s_e < s_{ds} \\ p_e & \text{if} \;\; s_{fl} < s_e < s_{sm} \\ p_e - 1 & \text{if} \;\; s_e < s_{fl} \end{cases}$$

where s_{ds}, s_{sm} and s_{fl} are the threshold values to identify discontinuities, smooth and flat solutions respectively. This procedure is carried out iteratively.

The performance of the p-adaptive method is illustrated for the solution of the subsonic flow past a cylinder. In the figure below, the p-distribution around the cylinder is given after applying the p-adaptive procedure three times. High accuracy is required in the regions near the wall and in the wake. This example shows that it is possible to choose an appropriate polynomial degree in each element and yet achieve the same accuracy that can be obtained with a high polynomial degree everywhere but using less computational resources. The automatic procedure reaches a stable p-distribution with $p_{min} = 1$ and $p_{max} = 4$. The accuracy of the solution measured through the entropy error on the wall is of the same magnitude of the accuracy obtained with a $p = 4$ solution everywhere $(||\epsilon(p_{max}) - \epsilon(1 \leq p \leq p_{max})||_{L_2} < 0.01)$ where ϵ is the L_2 norm of the difference

between the exact and the computed solution.

The efficiency of the p-adaptive procedure comes from the reduction in the number of operations required to solve the equations, but also the initial condition of each $1 \leq p \leq p_{max}$ simulation is a converged solution obtained with a lower degree $(1 \leq p \leq p_{max} - 1)$. However, the smaller CFL time restriction associated with p_{max} has to be imposed over all the domain, thus reducing the time step also in the regions with lower polynomial order. A possible improvement could be the application of a domain decomposition technique or variable timestepping to deal with different values of Δt through the domain.

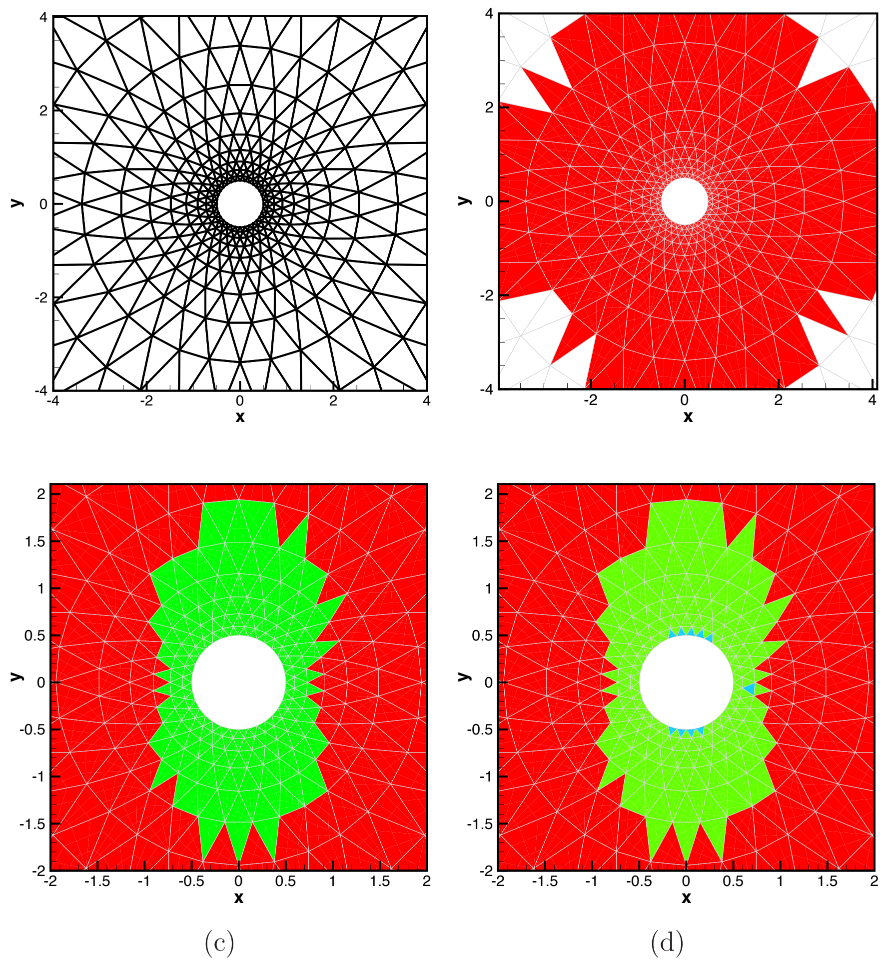

Figure 1: Polynomial degree distribution after applying the automatic p-adaption three times on a subsonic cylinder (a) p = 1 (white) (b) $1 \leq p \leq 2$ (red) (c) $1 \leq p \leq 3$ (green) (d) $1 \leq p \leq 4$ (blue).