ANISOTROPIC A-PRIORI ERROR ESTIMATES ON SURFACES

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Abstract.

A lot of practical problems are related to the resolution of Partial Differential Equations (PDEs) defined on surfaces embedded in a three dimensional space. In such cases the classical differential operators have to be suitably modified to recover tangential information (see [1]); likewise, the derivation of error estimators is usually not a trivial task, essentially due to the fact that these estimators should include the error due to the finite element approximation as well as to the fitting of the computational domain (see [3]).

Moving from the theory proved in [2], we propose an *anisotropic a-priori error estimator* to control the L^2-norm of the interpolation error associated with linear finite elements defined on surfaces. This new error estimator consists of two different contributions:

- an *almost best-approximation term*, typical of a finite element discretization;

- a *geometric error term*, related to the discretization of the surface.

Moving from to this estimator, we settle a metric-based anisotropic mesh adaptation procedure which essentially employs local operations (node smoothing, edge collapsing, edge splitting, edge swapping) to adapt the mesh. Since an anisotropic estimator takes into account the directional features of the solution at hand, we obtain adapted meshes whose elements are suitably oriented to match the intrinsic directionality of the function defined on the surface, and of the surface itself.

As expected the employment of anisotropic meshes leads to a remarkable improvement of the mesh adaptation procedure in terms of computational costs.

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