

EFFECTIVE APPLICATION OF THE EQUILIBRATED RESIDUAL METHOD IN ERROR ESTIMATION OF THE HP-APPROXIMATED 3D-BASED MODELS OF COMPLEX STRUCTURES

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Abstract. The paper recalls our previous theoretical results on [1] hierarchical modeling and adaptive analysis of complex structures (described with more than one mechanical model). Within such structures we apply 3D-based approach utilizing only three-dimensional degrees of freedom. The proposed estimation method [2] is based on the equilibrated residual one [3] and is applied to the assessment of the global, total and approximation errors. The global modeling error is obtained as a difference of the former two errors. The global modeling error estimate and the element contributions to it allow for the adaptive hierarchical modeling within first order shell, hierarchical shell and the corresponding transition (either shell-to-shell or solid to-shell) domains of complex structures. Both, the change of the mechanical model or q -adaptivity are possible, with q denoting the transverse order of approximation within hierarchical shell models. In the recalled approach also adaptive 2D, 3D or mixed (2D/3D) hp-approximations are possible in the shell, solid and transition zones of the complex structures, respectively, with h standing for the averaged element dimension and p denoting the longitudinal or three-dimensional order of approximation. The element contributions to the estimated global approximation error serve these two types of adaptivity.

In the above described context, it is very important to have the estimation method which can satisfy the specific needs of the complex structures 3D-based modeling and analysis and delivers sufficiently accurate estimated values of the global errors, and acceptable element contributions to them as well. In order to satisfy the mentioned needs we extend the existing algorithms of the equilibrated residual method [4, 5], applied so far to either three-dimensional elasticity [6] or conventional hierarchical shell models [7], onto the analysis of 3D-based first-order shell models [8, 9], 3D-based hierarchical shells and transition models as well. In particular we show unpublished technical details on how to apply this method to the 3D-based (constrained) shell model of the first order and the corresponding transition models as well. Such an application needs different equilibration procedure than the three-dimensional equilibration applied to 3D-elasticity (or hierarchical shells) and performed in the global directions. The adopted approach requires introduction of the local nodal directions and different treatment of the constrained and unconstrained ones.

In order to assess the quality of the equilibrated residual method we compare three versions of the method. The differences between these versions are visible while defining the element

local problems. Let us remember that the collection of solutions to such problems serves as a base for the assessment of the approximation of the exact global solution. In the first version we average the interelement stress fluxes, in the second one we perform linear (element vertex nodes) equilibration of these fluxes, while in the third one we constrain local problems at element vertices with global displacements. Then, for the most effective version (the third one), we perform unique parametric studies of the modeling, approximation and total error estimations. These studies include such important factors as: orders p and q , the element size h , the structure thickness, the applied mechanical model, the problem type, and mechanical complexity of the model. Our studies are completed with an analysis of the results. This analysis leads to practical hints concerning appropriate definitions of the local problems, so as to assure the most effective error estimation within complex structures.

REFERENCES

- [1] G. Zboinski, "Adaptive hpq finite element methods for the analysis of 3D-based models of complex structures. Part 1. Hierarchical modelling and approximations", *Computer Methods in Applied Mechanics and Engineering*, Vol. 199, pp. 2913-2940, (2010).
- [2] G. Zboinski, "Adaptive hpq finite element methods for the analysis of 3D-based models of complex structures. Part 2. A posteriori error estimation", Submitted to: *Computer Methods in Applied Mechanics and Engineering*.
- [3] M. Ainsworth, J. T. Oden, *A posteriori error estimation in finite element analysis*, Wiley & Sons, New York, 2000.
- [4] M. Ainsworth, J. T. Oden, "A posteriori error estimators for second order elliptic systems: Part 1. Theoretical foundations and a posteriori error analysis", *Computers Math. Applic.*, Vol. 25, pp. 101-113, (1993).
- [5] M. Ainsworth, J. T. Oden, "A posteriori error estimators for second order elliptic systems: Part 2. An optimal order process for calculating self-equilibrating fluxes", *Computers Math. Applic.*, Vol. 26, pp. 75-87, (1993).
- [6] M. Ainsworth, J. T. Oden, W. Wu, "A posteriori error estimation for hp approximation in elastostatics", *Appl. Numer. Math.*, Vol. 14, pp. 23-55, (1994).
- [7] J. T. Oden, J. R. Cho, "Adaptive finite element methods of hierarchical shell models for plate- and shell-like structures", *Comput. Methods Appl. Mech. Engrg.*, Vol. 136, pp. 317- 345, (1996).
- [8] G. Zboinski, "A posteriori error estimation for hp-approximation of the 3D-based first order shell model. Part I. Theoretical aspects", *Applied Mathematics Informatics and Mechanics*, Vol. 8 (1), pp. 104-125, (2003).
- [9] G. Zboinski, "A posteriori error estimation for hp-approximation of the 3D-based first order shell model. Part II. Implementation aspects", *Applied Mathematics, Informatics and Mechanics*, Vol. 8 (2), pp. 59-83, (2003).