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NEW BOUNDING TECHNIQUES FOR GOAL-ORIENTED ERROR ESTIMATION IN FE SIMULATIONS

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Abstract. In the context of finite element (FE) model verification, research and engineering activities focus on the development of robust goal-oriented error estimation methods designed to achieve strict and high-quality error bounds associated to specific quantities of interest. A general method [1,2,3] consists in using extraction techniques as well as robust global error estimation methods, and leads to the global solution of an auxiliary problem, also known as dual or adjoint problem. The derivation of accurate local error bounds entails a fine resolution of this auxiliary problem. Nevertheless, the classical bounding technique may provide low-quality error bounds on specific quantities of interest, particularly when the global estimated errors related to both reference (primal) and adjoint (dual) problems are mainly concentrated in disjoint regions. The main source of overestimation presumably stems form the Cauchy-Schwarz inequality, especially when the zone of interest is located far from the predominant contributions of the global estimate associated to reference problem. This observation has spurred the development of new bounding techniques able to circumvent, or at least alleviate, this serious drawback by optimizing the sharpness and practical relevance of the classical computed bounds.

In this work, we propose and analyze two new improved bounding techniques based on non-classical and innovative tools, such as homotheticity properties [4]. These techniques are carefully tailored for the derivation of inequalities between appropriate quantities over two homothetic domains contained in the whole structure. Such relations are based on Saint Venant's principle and seem to be limited to solely linear problems. The classical and enhanced techniques are combined with an intrusive approach (local refinement techniques) or a non-intrusive one (handbook techniques [5]) to get a reliable solution of the adjoint problem. Handbook techniques consist in enriching the approximate solution of the adjoint problem locally by introducing precomputed or analytical known handbook functions through the partition of unity method (PUM). Numerical examples are provided with comparative results between conventional and alternative bounding techniques within the linear elasticity framework. Various linear quantities of interest (such as the local average of a stress component, the pointwise value of a displacement component or a stress intensity factor) are considered to illustrate the effectivity of the two proposed techniques.

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