

ERROR ESTIMATION FOR THE CONVECTIVE CAHN – HILLIARD EQUATION

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Abstract. The Cahn–Hilliard phase-field (or diffuse-interface) model has a wide range of fluid applications where the interest is the modelling of phase segregation and evolution of multiphase flow systems. In order to capture the physics of these systems, diffuse-interface models presume a nonzero interface thickness between immiscible constituents, see [1]. The multiscale nature inherent in these models (interface thickness and domain size of interest) urges the use of space-adaptivity in discretization.

In this contribution we consider the a-posteriori error analysis of the convective Cahn–Hilliard [4] model for varying Péclet number and interface-thickness (diffusivity) parameter. The adaptive discretization strategy uses mixed finite elements, a stable time-stepping algorithm and residual-based a-posteriori error estimation [2, 5]. This analysis for the convective model forms a basic step in our research and will be helpful to the coupled Cahn–Hilliard/Navier–Stokes system [3] which is the desired model for future research.

REFERENCES

- [1] Anderson, D.M., McFadden, G.B. and Wheeler, A.A. Diffuse-Interface Methods in Fluid Mechanics. *Annu. Rev. Fluid Mech.*(1998) **30**:139–65
- [2] Bartels, S., Müller, R. A-posteriori error controlled local resolution of evolving interfaces for generalized Cahn–Hilliard equations. *Interfaces and Free Boundaries* (2010) **12**:45–73

- [3] Boyer, F., Lapuerta, C., Minjeaud, S., Piar, B. and Quintard, M. Cahn-Hilliard/NavierStokes Model for the Simulation of Three-Phase Flows. *Transport in Porous Media* (2010) **82**:463483
- [4] Kay, D., Styles, V. and Süli, E. Discontinuous Galerkin Finite Element Approximation of the Cahn–Hilliard Equation with Convection. *SIAM J. Numer. Anal.*(2009) **47**:2660–2685
- [5] van der Zee, K. G., Oden, J. T., Prudhomme, S. and Hawkins-Daarud, A. Goal-oriented error estimation for Cahn–Hilliard models of binary phase transition. *Numer. Methods Partial Differ. Equations* (2011) **27**:160–196