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INEXACT-HESSIAN-VECTOR PRODUCTS FOR EFFICIENT REDUCED-SPACE PDE-CONSTRAINED OPTIMIZATION

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Abstract. Partial differential equation (PDE) constrained optimization problems can be posed in the full-space or the reduced-space. In full-space formulations the PDE state variables — e.g. pressure and velocity for incompressible flows — are included as optimization variables, and the PDE becomes an explicit constraint in the optimization. In contrast, reduced-space formulations treat the state variables as implicit functions of the design variables: for a given set of design variables the PDE is solved for the states.

In practice, engineers often prefer reduced-space formulations. Reduced-space methods lend themselves to modularity, so implementation is typically easier than full-space methods. Unfortunately, conventional reduced-space optimization algorithms exhibit poor algorithmic scaling. For example, the computational cost of quasi-Newton methods is often proportional to the number of design variables. This scaling limits the number of design variables that can be considered.

Motivated by the above observations, we consider reduced-space inexact-Newton-Krylov (INK) algorithms, which offer the potential for design-dimension-independent algorithmic scaling. One of the challenges with reduced-space INK methods for PDE-constrained optimization is the efficient computation of Hessian-vector products needed by the Krylov solver. In particular, it is believed that these products must computed with high accuracy to avoid convergence difficulties. This accuracy requirement can render reduced-space INK methods orders of magnitude more expensive than full-space methods. In this paper, we argue that the Hessian-vector products can be computed inexactly provided an appropriate Krylov solver is adopted. Numerical examples demonstrate that the resulting reduced-space INK algorithms are competitive with their full-space counterparts.