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# HIERARCHICAL MODEL (HIMOD) REDUCTION FOR ADVECTION-DIFFUSION-REACTION PROBLEMS

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**Abstract.** The effective numerical description of challenging problems arising from engineering applications demands often the selection of an appropriate reduced - aka "surrogate" - model. The latter should result from a trade-off between reliability and computational affordability (see, e.g., [1, 8]). Different approaches can be pursued to set up the reduced model. In some cases, one can take advantage of specific features of the problem at hand for devising an effective *ad-hoc* model reduction. This is the case, for instance, of problems featuring a prevalent direction in the dynamics of interest, as in the modeling of the hemodynamics in arterial trees or of the hydrodynamics in a channel network. In this context, a possible approach is represented by the so-called *geometrical multiscale*, where dimensionally heterogeneous models are advocated for describing interactions at different scales: essentially, a lower dimensional (for instance, 1D) model is locally replaced by a higher dimensional (for instance, 3D) model to include local relevant transversal dynamics. This approach has been successfully applied both in hemodynamics (see, e.g., [3, 4]) and in river dynamics (see, e.g., [6, 5]).

As an alternative to the geometrical multiscale formulation, the so-called *hierarchical modeling* has been advocated in [2, 7]. The basic idea is to perform a classical finite element discretization along the mainstream direction of the problem at hand coupled with a modal decomposition for the transversal dynamics. The rationale behind this approach is that the transversal dynamics can be suitably captured by a few degrees of modal freedom. In addition, the dimenson of the modal discretization can be suitably adapted along the main direction, according to the local features of the transversal component of

the solution. This allows to improve hierarchically and adaptively the descripton of the local transversal dynamics, still in the context of a ”psychologically” 1D solver.

Comparison and coupling with the geometrical multiscale approach, effective criteria for the selection of the hierarchical modal basis relying upon a principal component analysis (and alternative to the ones introduced in [2, 7]), applications to real 3D problems (such as the circulatory system) are steps to be tackled in the development of an effective HiMod approach in engineering applications.

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