VI International Conference on Adaptive Modeling and Simulation
ADMOS 2013
J. P. Moitinho de Almeida, P. Díez, C. Tiago and N. Parés (Eds)

A VARIATIONAL FORMULATION OF DAMAGE FOR THERMO-VISCOELASTIC PROBLEMS IN LARGE STRAINS

A.E. SELKE*, L. STAINIER* AND E.A. FANCELLO†

* Institut de Recherche en Génie Civil et Mécanique – GeM (UMR CNRS 6183)
Ecole Centrale de Nantes
1 rue de la Noë, 44321 Nantes, France
e-mail: Augusto.Emmel-Selke@ec-nantes.fr
Laurent.Stainier@ec-nantes.fr

† Grupo de Analise e Projeto Mecânico (GRANTE)
Campus Univesitario Reitor João David Ferreira Lima – Trindade
88040-900 Florianopolis, Brazil
e-mail: fancello@grante.ufsc.br

Key words: Variational formulation, Damage, Thermo-viscoelasticity, Large strains.

Abstract. A variational formulation of damage for thermomechanically coupled problems is proposed. Based on variational methods for dissipative materials developed in the last few years (especially since [1]), this formulation allows for the accurate simulation of a wide variety of damage models, encompassing the behavior of various classes of polymers, the target application of the present paper.

A full development of the proposed formulation is presented in the first section. From an energy-like scalar-valued functional, the balance equations of the problem are derived, including the coupling terms between the different physics. The treatment of thermal effects follows [2]. A distinction between an internal and an external temperature allows for a factorization that, contrary to more classic formulations, keeps a symmetric structure to the problem. In order to describe rate dependence phenomena, the deformation energy potential is constructed from a generalized Kelvin-Voigt/Maxwell rheological model. Eigenvalues of strains are used as independent variables so that large strains are properly treated, following [3]. Having established the ingredients for a variational formulation of thermo-viscoelasticity, we move on to include damage effects. After a brief description of the choices made in [4] for the modeling of low cycle metal fatigue, a general framework for the inclusion of different damage models is presented.

In the second section, different damage models are applied. Although the framework we present is sufficiently general to consider anisotropic models, only isotropic examples of damage are presented in this article. The simplest case, where a scalar damage variable acts only upon the elastic part of the deformations, is presented first. We then apply the damage variable to both elastic and viscous parts of the deformation, which yields a more complex behavior. The possibilities of developing even more complex damage models, with different

damage variables acting upon each rheological Maxwell branch, is also discussed – it should be noted, though, that such models bring about increasingly tougher challenges in parameter identification. Different damage evolution laws are tested to show the versatility of the formulation.

The third and final section consists of the validation of the proposed model for the simulation of some characteristic phenomena of polymer damage behavior. A final discussion of further possible applications of the formulation concludes the present article.

REFERENCES

[1] Ortiz, M. and Stainier, L. The variational formulation of viscoplastic constitutive updates. *Computer Methods in Applied Mechanics and Engineering* (1999) **171**:419-444.

[2] Yang, Q., Stainier, L. and Ortiz, M. A variational formulation of the coupled thermo-mechanical boundary-value problem for general dissipative solids. *Journal of the Mechanics and Physics of Solids* (2006) **54**:401-424.

[3] Fancello, E., Ponthot, J-P. and Stainier, L. A variational formulation of constitutive models and updates in nonlinear finite viscoelasticity. *International Journal for Numerical Methods in Engineering* (2006) **65**:1831-1864.

[4] Kintzel, O. and Mosler, J. An incremental minimization principle suitable for the analysis of low-cycle fatigue in metals: A coupled ductile-brittle damage model. *Computer Methods in Applied Mechanics and Engineering* (2011) **200**:3127-3138.