VI International Conference on Adaptive Modeling and Simulation
ADMOS 2013
J. P. Moitinho de Almeida, P. Díez, C. Tiago and N. Parés (Eds)

PROBABILITY AND VARIANCE-BASED STOCHASTIC DESIGN OPTIMIZATION OF A RADIAL COMPRESSOR CONCERNING FLUID-STRUCTURE INTERACTION

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Key words: robust design optimization, robustness evaluation, reliability analysis, fluid-structure interaction, surrogate models, adaptive design of experiment, importance sampling, directional sampling

Abstract. Since the engineering of turbo machines began the improvement of specific physical behaviour, especially the efficiency, has been one of the key issues. However, improvement of the efficiency of a turbo engine, is hard to archive using a conventional deterministic optimization, since the geometry is not perfect and many other parameters vary in the real approach.

In contrast, stochastic design optimization is a methodology that enables the solving of optimization problems which model the effects of uncertainty in manufacturing, design configuration and environment, in which robustness and reliability are explicit optimization goals. Therein, a coupling of stochastic and optimization problems implies high computational efforts, whereby the calculation of the stochastic constraints represents the main effort. In view of this fact, an industrially relevant algorithm should satisfy the conditions of precision, robustness and efficiency.

In this paper an efficient approach is presented to assist reducing the number of design evaluations necessary, in particular the number of nonlinear fluid-structure interaction analyses. In combination with a robust estimation of the safety level within the iteration and a final precise reliability analysis, the method presented is particularly suitable for solving reliability-based structural design optimization problems with ever-changing failure probabilities of the nominal designs.

The applicability for real case applications is demonstrated through the example of a radial compressor, with a very high degree of complexity and a large number of design parameters and random variables.

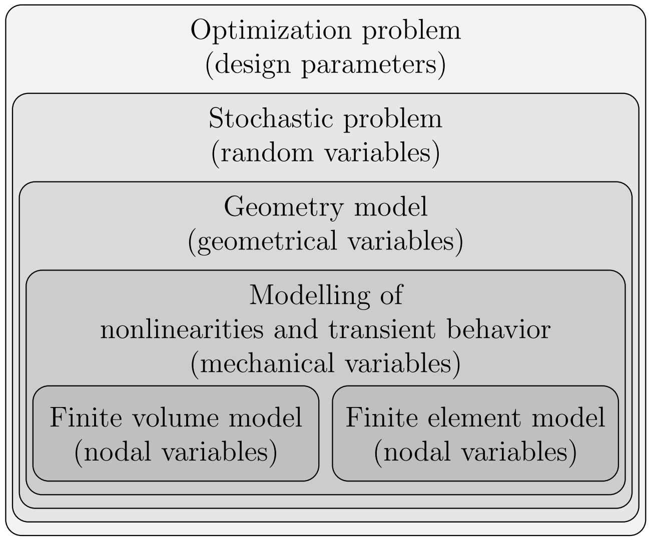

Figure 1: Coupled numerical models and different variable spaces of a stochastic design optimization of a fluid-structure interaction analysis based on a parametric geometry model (according Chateauneuf, 2008).

1 INTRODUCTION

1.1 Stochastic design optimization

In engineering problems, randomness and uncertainties are inherent and may be involved in several stages, for example in the system design with material parameters and in the manufacturing process and environment. Stochastic optimization, also referred to as reliability-based and variance-based optimization is known as the most adequate and advantageous methodology for system or process design and aims at searching for the best compromise design between design improvement and robustness or reliability assurance. Herein, the optimization process is carried out in the space of the design parameters and the robustness evaluation and reliability analysis are performed in the space of the random variables. Consequently, during the optimization process the design variables are repeatedly changed, whereby each design variable vector corresponds to a new random variable space. Therefore usually, a high number of numerical calculations are required to evaluate the stochastic constraints at every nominal design point. This repeated search becomes the main problem, especially when numerical nonlinear multi-domain simulations and CAD models are involved.

Unfortunately, in real case applications of the virtual prototyping process, it is not always possible to reduce the complexity of the physical models to obtain numerical models which can be solved quickly. Although progress has been made in identifying numerical methods to solve stochastic design optimization problems and high performance computing, in cases such as those that have several nested numerical models, as shown in Fig. 1, the actual costs of using these methods to explore various model configurations for practical applications is too high. Therefore, methods for efficiently solving stochastic optimization problems based on the introduction of simplifications and special formulations for reducing the numerical efforts are required. Note: an extended version of this paper is published in Roos et al. (2013).

1.2 Application to aerodynamic optimization

In comparative studies on the application of the deterministic optimization for aerodynamic optimization (see e.g. Sasaki et al., 2001, Shahpar, 2000) usually stochastic programming algorithms or response surface methods (see e.g. Pierret and van den Braembussche, 1999) are used in turbomachinery design, for example in the development of engine components, such as at Vaidyanathan et al. (2000). In Shyy et al. (2001) a comprehensive overview is represented.

Another very comprehensive study of the use of the combination of genetic algorithms and neural networks for two-dimensional aerodynamic optimization of profiles is presented in Dennis et al. (1999) combine a genetic algorithm with an gradient-based optimization method.

Furthermore, an increasing application of stochastic analysis on turbo machinery (e.g. at Garzon, 2003, Garzon and Darmofal, 2003, Lange et al., 2010, Parchem and Meissner, 2009) underlines the importance of integrating the uncertainty analysis into the aerodynamic design process.

2 RELIABILITY AND VARIANCE-BASED DESIGN OPTIMIZATION

2.1 Deterministic optimization

Optimization is defined as a procedure to achieve the best outcome of a given objective function while satisfying certain restrictions. The deterministic optimization problem

$$
\begin{aligned}
f(d_1, d_2, \dots d_{n_d}) &\rightarrow \min \\
e_l(d_1, d_2, \dots d_{n_d}) &= 0;\ l = 1, n_e \\
u_m(d_1, d_2, \dots d_{n_d}, \boldsymbol{\gamma}) &\geq 0;\ m = 1, n_u \\
d_{l_i} &\leq d_i \leq d_{u_i} \\
d_i &\in [d_{l_i}, d_{u_i}] \subset \mathbb{R}^{n_d}
\end{aligned}
\tag{1}
$$

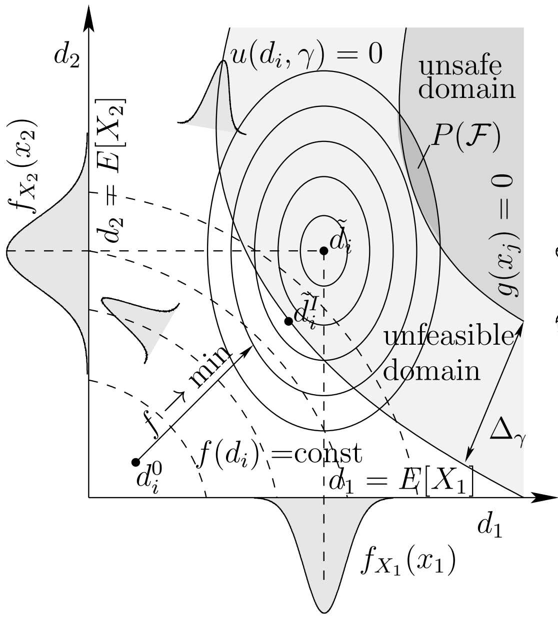

Figure 2: Different solution points $\tilde{d}_i^I$ or $\tilde{d}_i$ as result of a deterministic vs. stochastic design optimization in the space of given randomly distributed design parameters.

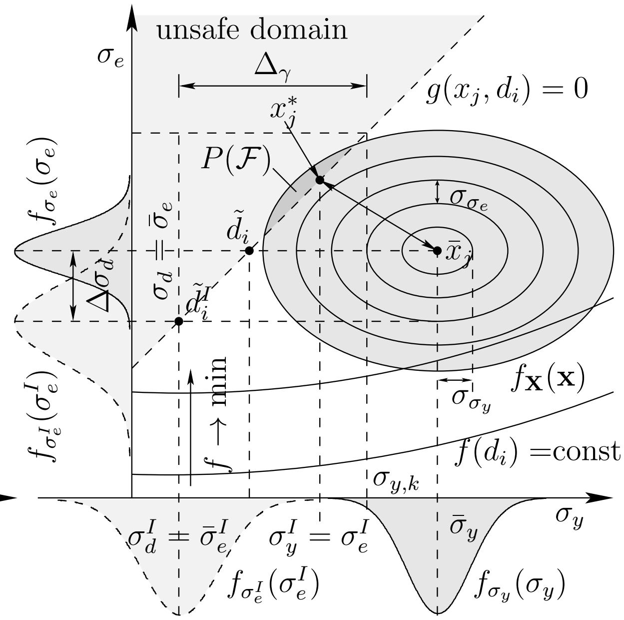

Figure 3: Comparison of the deterministic optimal point $\tilde{d}_i^I$ and the solution of a stochastic optimization $\tilde{d}_i$ with corresponding most probable failure point x_j^* in the space of the randomly distributed von Mises stress and the yield stress.

is defined by the objective function $f : \mathbb{R}^{n_d} \rightarrow \mathbb{R}$ subject to the restrictions, defined as equality and inequality constraints e_l and u_m. The variables $d_1, d_2, \ldots d_{n_d}$ are the optimization or design variables and the vector of the partial safety factors $\boldsymbol{\gamma}$ ensures the system or design safety within the constraint equations u_m, for example defining a safety distance $u(\mathbf{d}, \gamma) = y_g/\gamma - y_d \geq 0$ between a defined limit state value y_g and the nominal design value y_d of a physical response parameter $y = f(\mathbf{d})$. In structural safety assessment, a typical constraint for the stress is given as

$$u(\mathbf{d}, \gamma) = \sigma_{y,k}/\gamma - \sigma_d \geq 0 \tag{2}$$

ensuring the global safety distance

$$\Delta_\gamma = \sigma_{y,k} - \sigma_d = \sigma_{y,k} - \frac{\sigma_{y,k}}{\gamma} = \sigma_{y,k}\left(1 - \frac{1}{\gamma}\right)$$

between the defined quantile value $\sigma_{y,k}$ of the yield stress and the nominal design stress σ_d with the global safety factor γ, as shown in Fig. 3. Whereby, in the real approach with given uncertainties, σ_d corresponds to the mean von Mises equivalent stress $\bar{\sigma_e}$ at the current design point.

2.2 Stochastic chance-constrained optimization

Stochastic optimization algorithms use the quantification of uncertainties to produce solutions that optimize the expected performance of a process or design, ensuring the target variances of the model responses and failure probability. So, the deterministic optimization problem (1) can be enhanced by additional stochastic restrictions. For example, the expression for system reliability

$$1 - \frac{P(\mathcal{F})}{P^t(\mathcal{F})} \geq 0 \tag{3}$$

ensures that the probability of failure

$$P(\mathcal{F}) = P[\{\mathbf{X} : g_k(\mathbf{x}) \leq 0\}] = \int\limits_{g_k(\mathbf{x})\leq 0} \overset{n_r}{\cdots} \int f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{4}$$

cannot exceed a given target probability $P^t(\mathcal{F})$, considering the vector of all random influences

$$\mathbf{X} = [X_1, X_2, ..., X_{n_r}]^T \tag{5}$$

with the joint probability density function of the random variables $f_{\mathbf{X}}(\mathbf{x})$ and $k = 1, 2, ..., n_g$ limit state functions $g_k(\mathbf{x}) \leq 0$.

These enhancements of the problem (1) are usually referred to reliability-based design optimization, in which we ensure that the design variables d_i satisfy the given constraints (3) to some specified probabilities. As a consequence, now the design parameters

$$\mathbf{d} = E[\mathbf{X}] \tag{6}$$

are the means of the n_r random influences $\mathbf{X}$ with every changing density function during the optimization process. As a result of the random influences, now the objective and the constraints are non-deterministic functions.

2.3 Reliability analysis using adaptive response surface method

For an efficient probability assessment of $P(\mathcal{F})$, according to Eq. (4), a multi-domain adaptive design of experiment in combination with directional sampling (see e.g. Ditlevsen et al., 1990) is introduced in Roos (2011) to improve the accuracy and predictability of surrogate models, commonly used in applications with several limit state conditions. Furthermore, the identification of the failure domains using the directional sampling procedure, the pre-estimation and the priori knowledge of the probability level is no longer required. Therefore this adaptive response surface method is particularly suitable to solve reliability-based design optimization problems considering uncertainties with ever-changing failure probabilities of the nominal designs.

However, a reliability analysis method based on surrogate models, is generally suitable for a few random variables only. In case of the proposed probability assessment method, an efficient application is given up to $n_r = 10, ..., 25$, depending on the number of relevant unsafe domains. Therefore, a variance-based sensitivity analysis should be used to find a reduced space of the important random influences.

2.4 Global variance-based sensitivity analysis

In general, complex nested engineering models, as shown in Fig. 1 contain not only first order (decoupled) influences of the design parameters or random variables but also higher order (coupled) effects on the response parameter of a numerical model. A global variance-based sensitivity analysis, as introduced by Saltelli et al. (2008), can be used for ranking variables $X_1, X_2, \ldots, X_{n_r}$ with respect to their importance for a specified model response parameter

$$Y = f(X_1, X_2, \ldots, X_{n_r})$$

depending on a specific surrogate model $\tilde{Y}$. In order to quantify and optimize the prognosis quality of these meta models, in Most and Will (2008) and Most (2011) the so-called coefficient of prognosis

$$\mathrm{COP} = \left(\frac{\mathbf{E}[Y_{\mathrm{Test}} \cdot \tilde{Y}_{\mathrm{Test}}]}{\sigma_{Y_{\mathrm{Test}}} \sigma_{\tilde{Y}_{\mathrm{Test}}}} \right)^2 ; \quad 0 \leq \mathrm{COP} \leq 1 \tag{7}$$

of the meta model is introduced. In contrast to the commonly used generalized coefficient of determination R^2 based on a polynomial regression model, in Eq. (7) variations of different surrogate models $\tilde{Y}$ are analyzed to maximize the coefficient of prognosis themselves. This procedure results in the so-called meta model of optimal prognosis, used as surrogate model $\tilde{Y}$ with the corresponding input variable subspace which gives the best approximation quality for different numbers of samples, based on a multi-subset cross validation obtained by latin hypercube sampling (see e.g. Huntington and Lyrintzis, 1998).

The single variable coefficients of prognosis are calculated as follows

$$\mathrm{COP}_i = \mathrm{COP} \cdot \tilde{S}_{T_i} \tag{8}$$

with the total sensitivity indices

$$\tilde{S}_{T_i} = \frac{E(V(\tilde{Y}|\mathbf{X}_{\sim i}))}{V(\tilde{Y})} \tag{9}$$

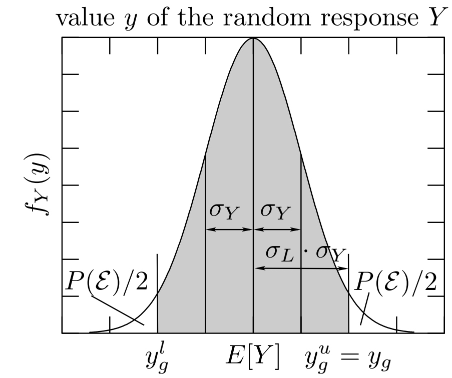

Figure 4: Relationship between density function $f_Y(y)$ of a model response, sigma level and exceedance probability, depending on chosen limit state conditions $y_g^{u,l}$.

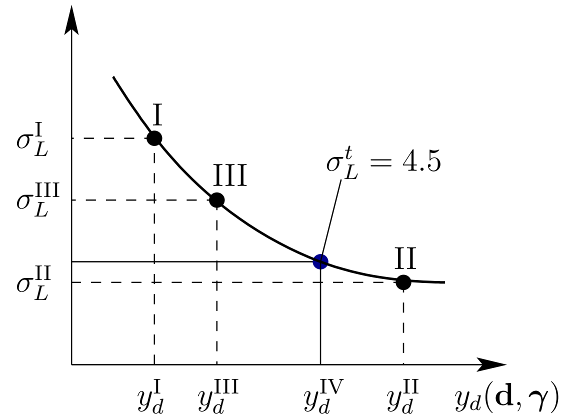

Figure 5: Convergence of a sequential stochastic chance-constrained optimization with successive interpolation of the nominal response limit y_d to ensure a target sigma level σ_L^t.

which have been introduced by Homma and Saltelli (1996), where $E(V(\tilde{Y}|\mathbf{X}_{\sim i}))$ is the remaining variance of $\tilde{Y}$ that would be left, on average, if the parameter of X_i is removed from the model. In Eq. (9) $\mathbf{X}_{\sim i}$ indicates the remaining set of input variables.

2.5 Probability estimation based on moments

For an accurate calculation of the reliability it would be interesting to expand the probability density function of the model responses about a critical threshold. Unfortunately, the density functions are unknown, especially close to the unsafe domain with high failure probability. Existing methods such as polynomial expansions, maximum entropy method or saddlepoint expansion, as reviewed in Hurtado (2008), are frequently used within the reliability-based structural optimization replacing the expensive reliability analysis.

A more simple, non-intrusive approach for a rough estimation of the failure probability is the calculation of the minimal sigma level σ_L for a performance-relevant random response parameter Y defined by an upper and lower limitstate value $y_g^{u,l} := \{Y|g(X) = 0\}$ as follows

$$E[Y] \pm \sigma_L \cdot \sigma_Y \overset{!}{\lessgtr} y_g^{u,l}$$

The sigma level can be used in conjunction with standard deviation to measure the deviation of response values Y from the mean $E[Y]$. For example, for a pair of

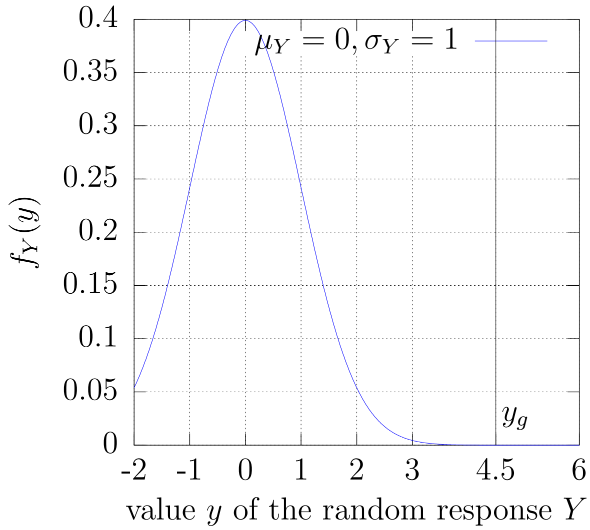

Figure 6: Gaussian density function $f_Y(y)$ of random response with upper specification limit $y_g := \{Y|g(X) = 0\}$.

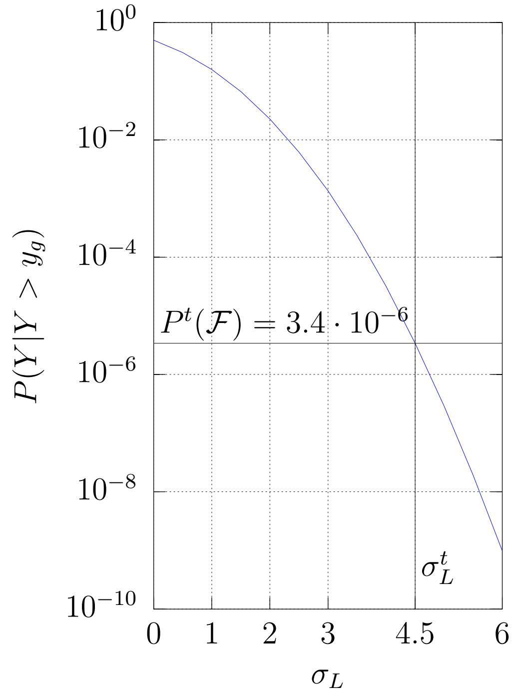

Figure 7: Sigma level variation and associated probability of failure (assumption: normal distribution for all important random responses).

quantiles (symmetrical case) and the mean value we obtain the assigned sigma level

$$\sigma_L = \frac{y_g - E[Y]}{\sigma_Y} \tag{10}$$

of the limit state violation, as explained in Fig. 4. Therewith, the non-exceedance probability results in

$$P(\mathcal{E}) = P(\{Y|Y \lesseqgtr y_g^{u,l}\}) = f(\sigma_L)$$

as a function of the sigma level, depending on the current distribution type of Y. In the same manner failure probability

$$P(\mathcal{F}) = P(\{Y|Y > y_g\}) = f(\sigma_L) \tag{11}$$

is given as a function of the sigma level. For example, assuming a normal distribution of the random response Y with $\mu_Y = 0$ and $\sigma_Y = 1$, as shown in Fig. 6, the failure probability is given as a nonlinear function

$$P(\mathcal{F}) = \Phi(-\sigma_L) = \Phi(-y_g)$$

of the sigma level, as illustrated in Fig. 7. Therewith, a probability of $P^t(\mathcal{F}) = 3.4 \cdot 10^{-6}$ is achieved when the performance target σ_L^t is 4.5 σ away from the mean value.

Other values of acceptable annual probabilities of failure $P^t(\mathcal{F})$ depending on the consequence of failure, significance warning or without warning before occurrence of failure and (non-)redundant structures can be found in engineering standards, e.g. in DNV (1992).

2.6 Methods solving stochastic optimization problems

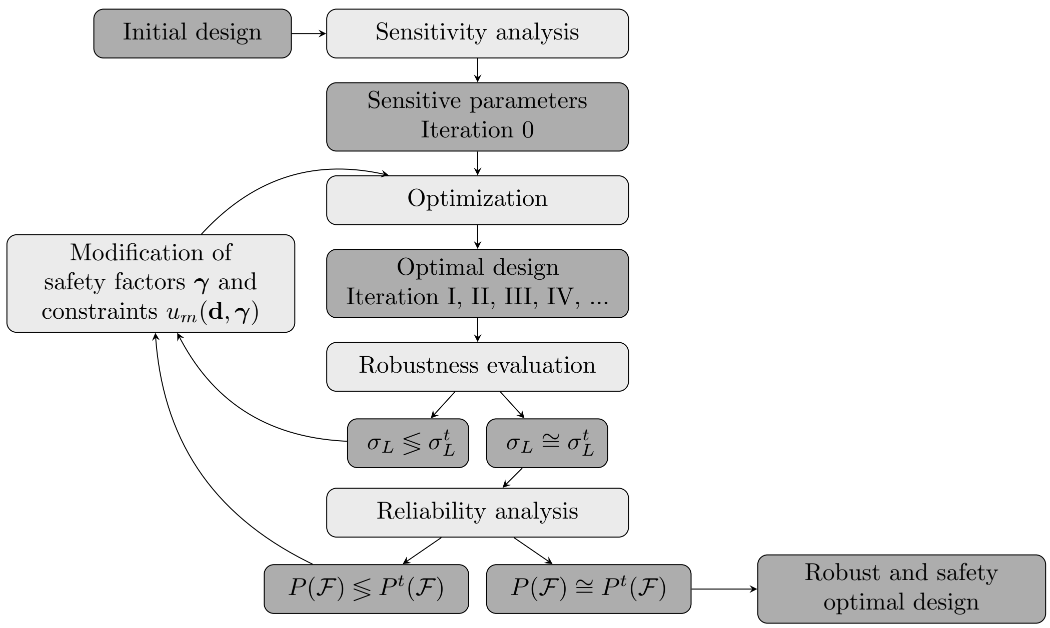

Figure 8: Basic concept of a decoupled loop of a reliability-based and variance-based stochastic design optimization using global variance-based sensitivity analysis and robustness evaluation to reduce the design parameter and random variable space.

In general, problem (1) to (6) is solved as a combination of a deterministic optimization in the n_d-dimensional design space and a stochastic analysis in the n_r-dimensional random space. Derivative-free global optimization methods are typically recommended to solve the sequential deterministic optimization problem, according to Eq. (1) for highly nonlinear numerical models, especially fluid-structure interaction models with probability-based constraints, whose objective and constraint function value may be computed with some noise or are non-computable in any design points.

Evolutionary computation, as a special class of global optimization strategies, imitates the natural processes like biological evolution or swarm intelligence. Based on the principle "survival of the fittest" a population of artificial individuals searches the design space of possible solutions in order to find a better solution for the optimization problem. In this paper an evolution strategy using a class of evolutionary algorithms is used. This strategy uses normally distributed mutations, recombination, selection of the best offspring individuals, and the principle of self-adaptation of strategy parameters, as described in Bäck (1995).

As an alternative derivative-free optimization method, especially useful for expensive numerical computations, we use the adaptive response surface methodology, as introduced in Etman et al. (1996), Toropov and Alvarez (1998), Abspoel et al. (1996), Stander and Craig (2002), Kurtaran et al. (2002).

Mainly, there are three methods for solving these kinds of coupled problems (1) to (6). The simplest and most direct solution method is a coupled approach in which a full reliability analysis is performed for every optimization function evaluation (see e.g. Choi et al., 2001). This involves a nesting of two distinct levels of optimization within each other, one at the design level and one at the reliability analysis level. This coupled procedure leads in general to an inefficient double loop with a large number of design evaluations.

The single-loop method (see e.g. Kharmanda et al., 2002) simultaneously minimizes the objective function and searches for the β-point, satisfying the probabilistic constraints only at the optimal solution, but needs a sensitivity analysis to analytically compute the design gradients of the probability constraint.

An alternative method, used in the following, is the sequential approach (see e.g. Chen et al., 2003). The general concept is to iterate between optimization and uncertainty quantification, updating the constraints based on the most recent probabilistic assessment results, using safety factors or other approximation methods. This effective iterative decoupled loop approach can be enhanced by updating the constraints during the internal optimization using sigma levels and statistical moments

$$\frac{\sigma_{L_k}}{\sigma_L^t} - 1 \geq 0;\ \sigma_{L_k} = \frac{y_{gk} - E[Y_k]}{\sigma_{Y_k}};\ k = 1, n_g$$

in place of the exceedance probability of the Eq. (3). Essentially, by means of transformation in Eq. (11) of the probability-based highly nonlinear and non-differentiable constraints to linear ones, these functions may be more well conditioned for the optimization approach and we can expect a better performance of the solution process. Of course, the transformation in Eq. (11) can only be used as a rough estimation of the safety level and we have to calculate the probabilities of failure using the

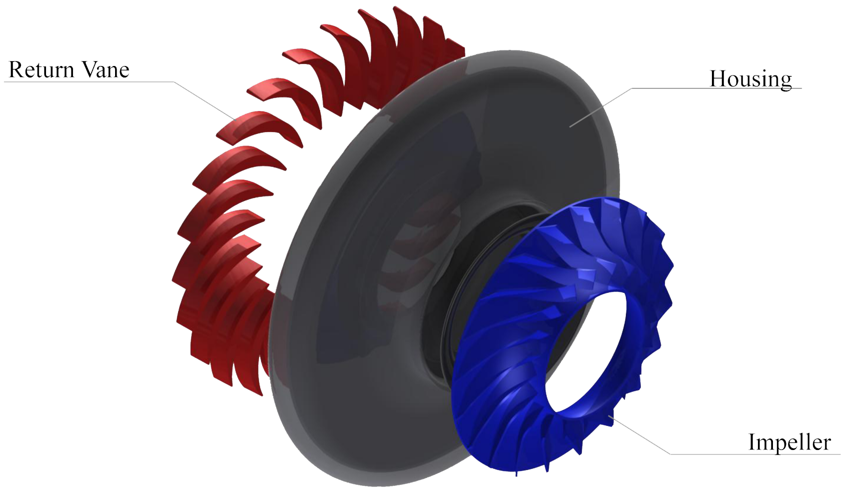

Figure 9: Parametric CAD model of a one stage radial compressor, consisting of a impeller and returnvane

reliability analysis, at least at the iteration end.

As shown in Fig. 8, in the initial iteration step a variance-based sensitive analysis identifies the most important multivariate dependencies and design parameters. After this, the deterministic optimization step results in an optimal solution for which the sigma level is calculated using a robustness evaluation, based on a latin hypercube sampling. The size of violation of the target sigma level is used to interpolate the constraints using modified safety factors. Whereby, as an important fact, the interpolation order increases continuously with each iteration step, so in practice three or four iteration steps may meet our optimization requirements in terms of robustness and safety. Fig. 5 shows a typical convergence of a sequential stochastic chance-constrained optimization.

Furthermore, the optimization steps and the final reliability analysis run mostly efficiently in the space of the current significant parameters. So every size of problem definition (number of design and random parameters) is solvable within all sigma levels.

The following numerical example with a very high degree of complexity is given to demonstrate the solving power of this sequential stochastic chance-constrained optimization by adapting the constraint $u_m(\mathbf{d}, \gamma)$ depending on interpolated nominal response values y_d.

3 NUMERICAL EXAMPLE

3.1 Fluid-structure interaction model

The stochastic optimization method presented here is applied to a CAD and CAE parameter-based design optimization of a radial compressor shown in Fig. 9, including material, process and geometry tolerances. In the example presented the target of the optimization process is to maximize the efficiency of the turbine engine with respect to a limitation of the maximal v. Mises stress. Additional constraints are defined by resonance of any eigen frequency with the rotational velocity of the rotor. In total 36 optimization parameters and 49 random influences are defined.

The Calculations were done with the software ANSYS Workbench and the probabilistic and optimization tasks were performed with the optiSLang software package.

As the method was already explained in Sec. 2, the results of the example are summarized. For a extended version see Roos et al. (2013).

3.2 Decoupled stochastic optimization loop

Through the sensitivity analysis the design parameters were reduced to 10 design variables with a relevant coefficient of optimal prognosis. The mean efficiency of the initial radial compressor was 86%. The best design of the latin hypercube sampling with an efficiency of 88.9% is used as start design of an evolutionary optimization based on the surrogate model of the meta model of optimal prognosis and gives with one additional design evaluation an efficiency of 89.3%. The distance of the design stress to the 5% quantile of the yield strength is a result of the first global safety factor of $\gamma^I = 1.5$ of the first iteration step. The target sigma level is $\sigma_L^t = 4.5$ to ensure a probability of failure $P(\mathcal{F}) = 3.14 \cdot 10^{-6}$. In the following, only the results of each iteration are shown in the Tab. 1.

Of course, the probability levels of violation of the limit state conditions or of the initial efficiency are only a rough estimation and at least a reliability analysis of the final design is recommended, especially for small probability levels. With the identification of the random sub domain directional sampling on adaptive moving least square is used for reliability analysis (see Sec. 2.3). The moving least square approximation is based on $N = 56$ design evaluations of an adaptive D-optimal design of experiment, as shown in Figs. 10 and 11. The assigned failure probability $\bar{P}(\mathcal{F}) = 2.5 \cdot 10^{-6} \leq P^t(\mathcal{F}) = 3.4 \cdot 10^{-6}$ indicates an optimized six sigma design.

Finally, the Figs. 12 and 13 show the flow along the return vane blades. It is distinctly and visibly how the separations have been reduced in the optimized design and a more uniform flow is present.

Safety factor γ^{i}, sigma level σ_L^{i}, σ_d^{i} and efficiency η_i				
i	γ^{i}	σ_L^{i}	σ_d^{i}	η^i
0	2.4	-	$1.27 \cdot 10^8$	86%
1	1.5	5.13	$1.67 \cdot 10^8$	90.5%
2	1.32	3.6	$1.75 \cdot 10^8$	90.8%
3	1.426	4.1	$1.71 \cdot 10^8$	90.0%
4	1.46	4.48		91%

Table 1: Results for each iteration step i.

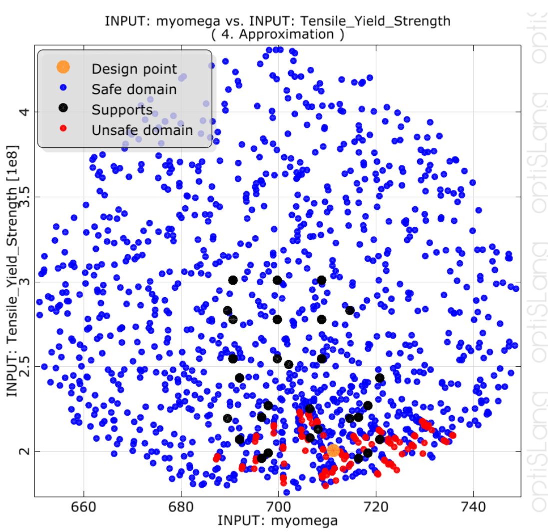

Figure 10: Anthill plot of the analyzed $N = 56$ design evaluations of the reliability analysis within iteration step IV between efficiency η and yield stress σ_y.

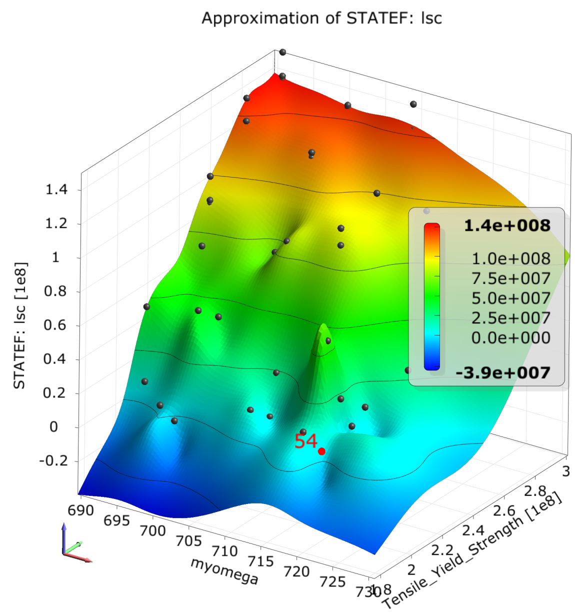

Figure 11: Response surface plot of the reliability analysis design IV.

4 CONCLUSIONS

In this paper an efficient iterative decoupled loop approach is provided for reducing the necessary number of design evaluations. The applicability of this method for

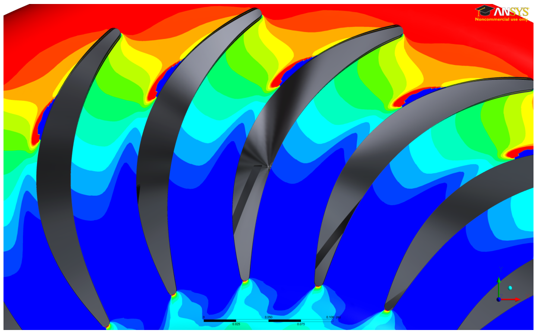

Figure 12: Flow angle of the initial design at the returnvane blades with separations along the blades

Figure 13: Flow angle of the optimized design at the returnvane blades with a much more uniform flow

real case applications is demonstrated for a radial compressor. Using the approach presented, it is possible to improve the efficiency by about 5%. In addition we obtain an optimized design which is insensitive to uncertainties and considers the target failure probability.

5 ACKNOWLEDGEMENT

This project is kindly supported through the program “Internal Research Funding”, funded through the Niederrhein University of Applied Sciences. The authors would like to express their thanks to Johannes Einzinger of the ANSYS Germany GmbH for his assistance in formulating the FSI design problem and to Ulrike Adams and Daniela Ochsenfahrt of the DYNARDO GmbH for their support within the method implementation into the optiSLang software package and the collaborative work.

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