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ANISOTROPIC ADAPTIVE NEARLY BODY-FITTED MESHES FOR CFD

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Abstract. This paper presents a numerical study of a recent technique that consists in modeling embedded geometries by a level-set representation in combination with local anisotropic mesh refinement. This method proves beneficial in CFD simulations involving complex geometries, as it suppresses the need for the tedious process of body-fitted mesh generation, without altering the finite element formulation nor the prescription of boundary conditions. The first part of the study deals with a simple Laplace problem featuring a planar interface on which a Dirichlet boundary condition is imposed. It is shown that the appropriate amount of local isotropic refinement yields the optimal convergence, unlike uniform refinement. Anisotropic refinement further ensures geometric convergence and limits the growth of the number of unknowns. The second part deals with the adaptive strategy for CFD problems. We show that the methodology yields accurate flow solutions, despite very limited user interaction.

1 INTRODUCTION

Because of the increasingly complex geometries involved in flow problems of industrial relevance, numerical methods based on unstructured meshes have become popular in CFD. However, the corresponding meshing methods require a high-quality CAD description of the geometry, which is not part of the traditional workflow in fields like architecture or medicine. Many professionals also lack the expertise required to build appropriate meshes for flow problems. Nevertheless, recent progresses in meshing technology could overcome these barriers.

In this work, we use anisotropic adaption to generate a nearly body-fitted mesh. The mesh is locally refined depending on a level-set function that describes the geometry without resorting to a CAD model [1, 2]. Dirichlet boundary conditions can then be imposed in a strong manner by node collocation, just as with classical body-fitted meshes. Unlike other treatments of embedded geometries, this technique only requires a standard finite element formulation, without basis enrichment or Lagrange multipliers that alter its numerical properties.

2 ADAPTIVE STRATEGY FOR NEARLY BODY-FITTED MESHES

A metric-based anisotropic mesh adaptation procedure is performed. It generates a uniform unit mesh [3] in a prescribed Riemannian metric space that corresponds to an anisotropic adapted mesh in the Euclidean space. Anisotropic mesh adaptation is performed in the vicinity of the interface Γ described by the level-set function $\phi(\mathbf{x})$, i.e. in a band $\{\mathbf{x} \text{ s.t. } |\phi(\mathbf{x})| \leq E\}$ of thickness $2E$ around Γ. With a linear discretization, the approximation error on the level-set function $\phi(\mathbf{x})$ is of second order. An appropriate metric field $\mathcal{M}$ can thus be constructed from the gradient vector $\nabla\phi(\mathbf{x}) = (\phi_x\ \phi_y\ \phi_z)^T$ and the Hessian matrix $\mathcal{H}(\phi(\mathbf{x}))$ of $\phi(\mathbf{x})$. More details about the construction of the metric can be found in [4].

In a first step, we apply this method to an academic 2D Laplace problem in a square with an embedded planar surface [5]. The solution is compared to the results obtained on anisotropic meshes with results obtained on uniform refined meshes and isotropic adaptive refined meshes (see Fig. 1). We show that an appropriate level of local refinement around the geometry recovers the optimal grid convergence rate for the solution, whereas uniform refinement yields first-order convergence as can be seen in the left plot of Fig. 2.

We also show in the right plot of Fig. 2 that controlling the anisotropic character of the adaption further enables the error of the geometrical discretization to decrease at optimal rate, which is not the case for isotropic refinement. This affects particularly the computation of integral quantities, such as lift and drag in CFD. Anisotropic adaptive refinement also slows down the growth of the number of unknowns, which limits the computational overhead.

3 ADAPTIVE MESHES FOR CFD

The adaptive strategy for CFD combines the presented nearly body-fitted adaptive mesh strategy with an iterative anisotropic adaption to the flow solution. A second mesh metric is constructed by calculating a scaled eigenspace of the Hessian matrix of the norm of the velocity. Indeed, as we are using linear finite element interpolation for the solution of the Navier-Stokes equations, the interpolation error is equivalent to second order derivatives and it has been shown that a large proportion of the discretization error is governed by this error indicator. This second mesh metric is then intersected with the level-set based anisotropic mesh metric.

We present two incompressible flow problems involving respectively a cylinder and more complex geometry case, namely an array of cylinders. The overall approach for the CFD

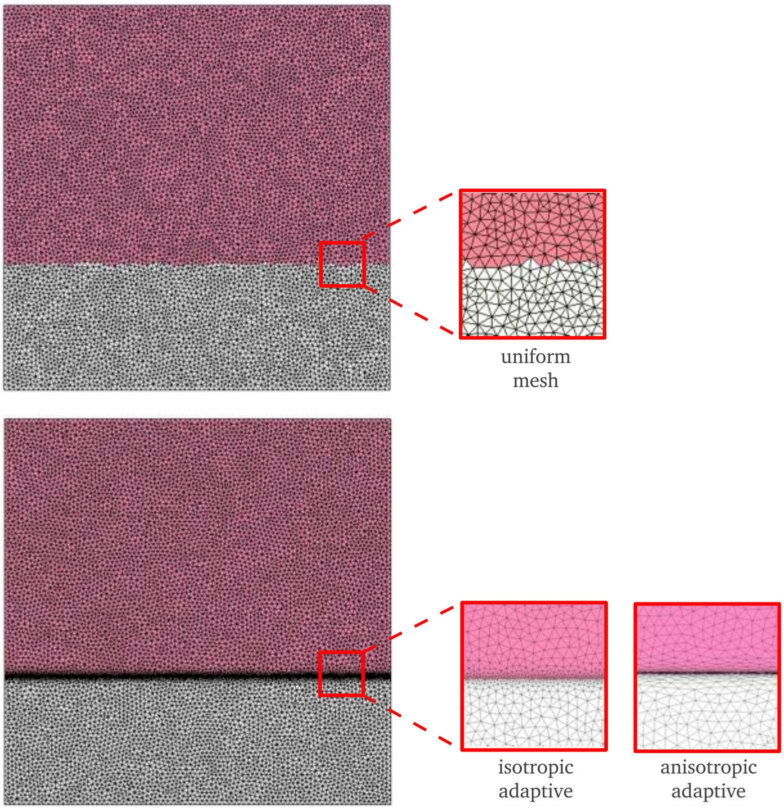


Figure 1: Uniform refined mesh and adaptive refined meshes

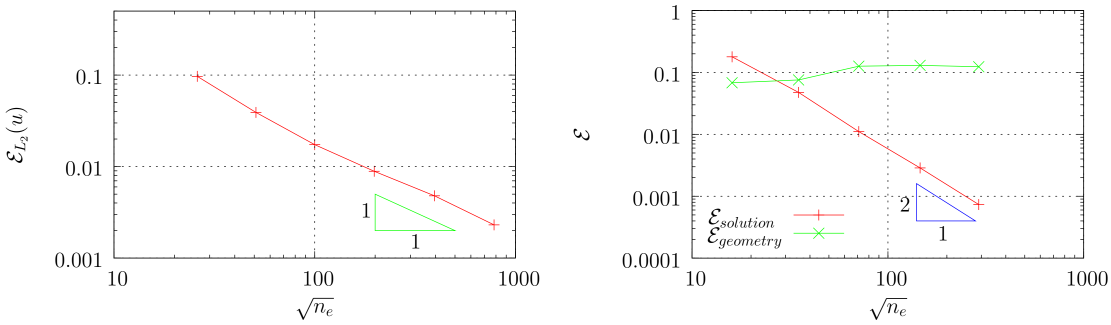


Figure 2: Solution error on uniform refined meshes (left) and geometry error on isotropic adaptive refined meshes (right)

problems can be explained as follows: the problem is first solved on a very coarse mesh with anisotropic elements in the vicinity of the interfaces. The mesh is then successively adapted to both the geometry and to the flow field. For the unsteady case, the solution at time steps which correspond to maximal values of the lift coefficient is used for iteratively adapting the mesh (see Fig. 3).

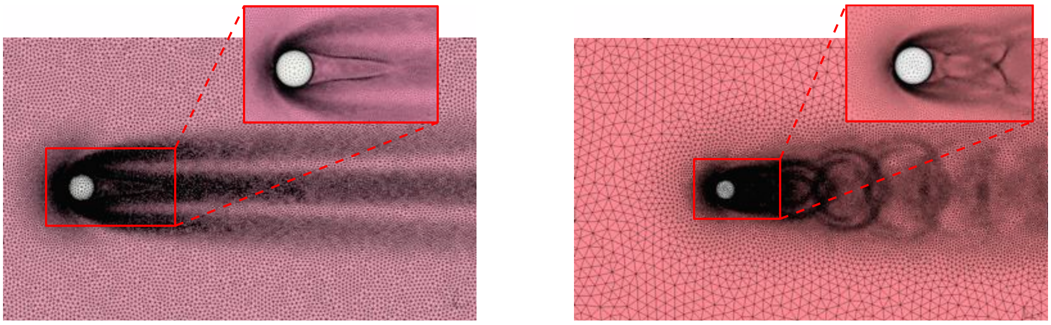


Figure 3: Adaptive mesh for steady flow (left) and unsteady flow (right) over a 2D circular cylinder

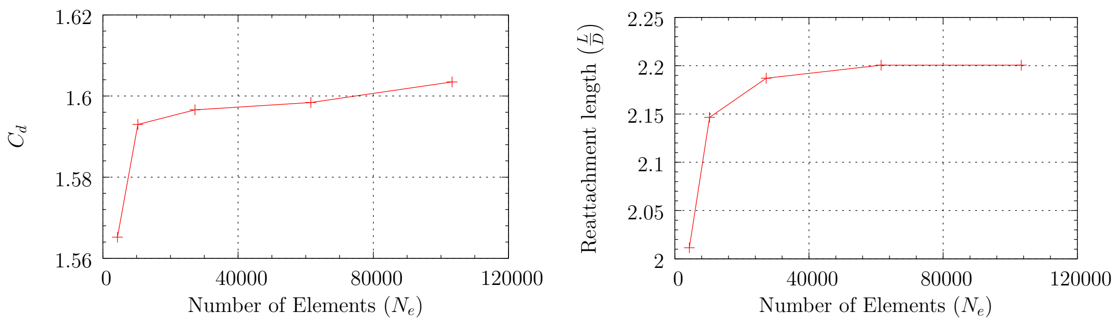


Figure 4: Convergence of drag coefficient and reattachment length at $Re = 40$

While both drag and reattachment length in the steady flow over the cylinder converge to the expected value in Fig. 4, the unsteady case also demonstrates the accuracy of the method as can be seen in Fig. 5.

Concerning the application to a complex geometry, we consider the benchmark described in Geller et al. [6]. The solution is in good agreement with the reference results.

4 CONCLUSION

The use of the standard finite element solver for solving CFD problems on "nearly body-fitted meshes" proves that the optimal rate of convergence can be obtained, and that the methodology yields accurate flow solutions, despite very limited user interaction.

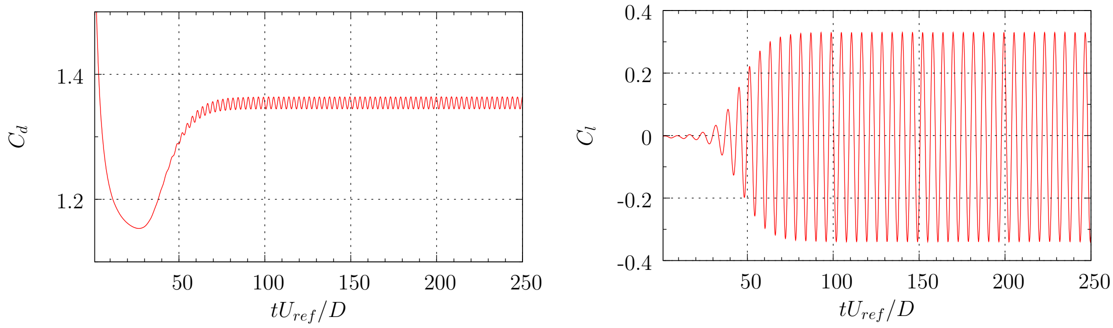


Figure 5: Lift and drag coefficients at $Re = 100$

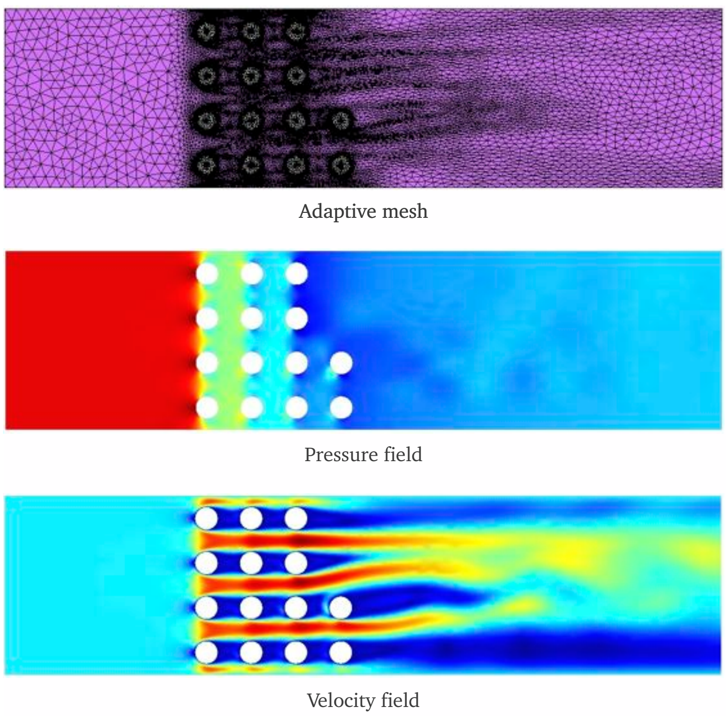
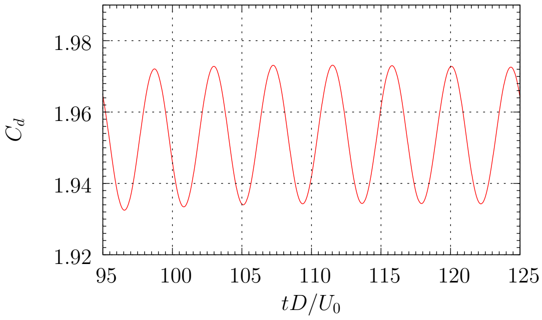


Figure 6: Computational mesh for flow over array of cylinders at $Re_E = 200$



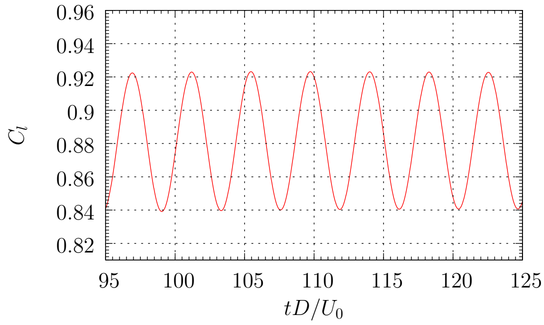


Figure 7: Lift and drag coefficients at $Re_E = 200$

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