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A POSTERIORI ERROR ESTIMATION OF TARGET CONTROL PROBLEMS: WEAK FORMULATION OF INEQUALITY CONSTRAINTS

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Abstract. This contribution considers the steering of movements of a mechanical system from an initial state to a target state (target control). Suitable FE-approximations for the state and control variables are discussed along with a goal-oriented a posteriori estimate of the discretization errors.

1 EXTENDED ABSTRACT

We shall in this contribution consider optimal control problems concerning the steering of motion of a mechanical system from an initial state to a target state (target or trajectory control). The motion of the system depends on forces acting as controls and is represented by a set of ordinary differential equations with load terms. By considering the equations of motion and the relevant kinematic and control limitations, a constrained optimization problem can be formulated where the control forces are sought to minimize a chosen objective function, such as the energy consumption or the deviation from desired trajectory, while reaching the defined target. As a numerical example we consider the search for an optimal brake-turning strategy for a vehicle manoeuver [4].

A discretization of finite element type in time is introduced, whereby approximations for the state (coordinates and velocities) and the control (external forces) variables are introduced. The optimality conditions are expressed in weak form, in particular, the inequality constraints are enforced weakly, whereby to what extent the inequality constraints are satisfied depend on the chosen discretization. The subject of the present work is to determine the error in the approximate solution compared to the exact solution, in particular with respect to how well the discrete solution satisfies inequality constraints and target conditions.

To this end, we employ a posteriori error estimates based on the pertinent dual problem (from linearization of the weak form) with some modification, whereby discretization errors in both state and control variables can be estimated in terms of chosen goal quantities. The sources of errors can be traced to specific regions of the state and control time-meshes, which can be used in an adaptive mesh-refinement procedure since the control and state variables are discretized separately. Earlier work on a posteriori error estimation for optimal control problems have been based on the Heidelberg approach[1, 4], whereas the present contribution will use our previous work in error control for parameter identification problems based on a tangent form of the dual problem[2].

To illustrate the problem setting, consider the example[3] of a double pendulum representing simple mechanical system. The pendulum is to be lifted from vertical hanging to a horizontal straight position using minimal control force without violating anthropomorphic constraints (the opening angle of the middle joint must be between 0 and 135 degrees) and control constraints (the control variables are restricted by maximum and minimum values). The solution algorithm is based on a nested format with a relaxation of the constraints.

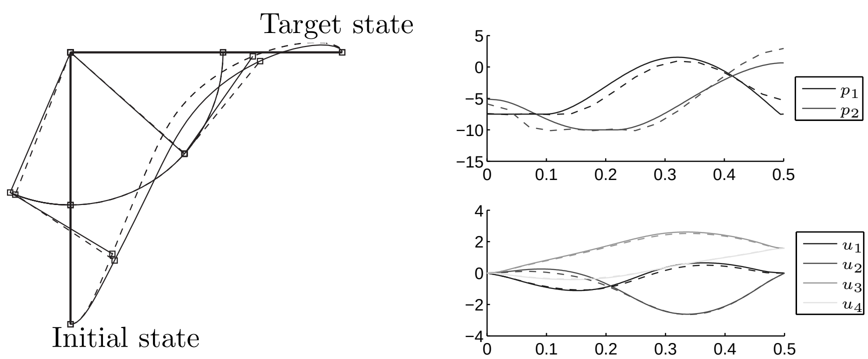

Figure 1: Left: Plot of the arm from vertical hanging to a horizontal straight position. Right: Comparison of the solved state (u_1-u_4) and control (p_1-p_2) variables using fine (dashed line) and coarse discretization (solid line).

The numerical example indicate that a discretization error in the control variable arises in order to "compensate" for discretization errors in the solution of the equations of motion in order to reach the target state.

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