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A FRAMEWORK FOR ROBUST A POSTERIORI ERROR CONTROL IN UNSTEADY NONLINEAR ADVECTION-DIFFUSION PROBLEMS

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Abstract. We derive a framework for a posteriori error estimates in unsteady, nonlinear, possibly degenerate, advection-diffusion problems. Our estimators are based on a space-time equilibrated flux reconstruction and are locally computable. They are derived for the error measured in a space-time mesh-dependent dual norm stemming from the problem and meshes at hand augmented by a jump seminorm measuring possible nonconformities in space. Owing to this choice, a guaranteed and globally efficient upper bound is achieved, as well as robustness with respect to nonlinearities, advection dominance, domain size, final time, and absolute and relative size of space and time steps. Local-in-time and in-space efficiency is also shown for a localized upper bound of the error measure. In order to apply the framework to a given numerical method, two simple conditions, local space-time mass conservation and an approximation property of the reconstructed fluxes, need to be verified. We show how to do this for the interior-penalty discontinuous Galerkin method in space and the Crank–Nicolson scheme in time. Numerical experiments illustrate the theory. More details on the analysis and results can be found in [1].

REFERENCES

[1] V. Dolejší, A. Ern and M. Vohralík, *A framework for robust a posteriori error control in unsteady nonlinear advection-diffusion problems. SIAM J. Numer. Anal.*, in press, 2013; see also HAL Preprint 00652979 v2, 2012.