# AIRFOIL OPTIMIZATION WITH TRANSITION CURVE AS OBJECTIVE FUNCTION 

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#### Abstract

The present work describes the design optimization of a low Reynolds number high lift airfoil where the objective function is that curve defined by the lift coefficient variation with the boundary layer transition position along the upper and lower surfaces of the airfoil. An aerodynamic shape optimization program using XFOIL as the solver, a viscous/inviscid two-dimensional panel method formulation code, and a sequential quadratic programming optimization routine, solves a minimization problem to determine the optimal airfoil geometry which minimizes the difference between its lift coefficient versus transition position curves and the specified objective curves while subject to geometric constraints and constant product of Reynolds number with the square root of lift coefficient for a given interval of lift coefficient values. The airfoil design variables are B-spline control points which define the airfoil camber line and the airfoil thickness distribution. A case study is presented for an airfoil design suitable for a long endurance UAV demonstrating the capability of the approach in producing an optimized design. Comparisons with other objective functions are also shown.


## 1 INTRODUCTION

In all conventional subsonic aircraft with medium/high aspect ratio wings (>6) the major single contribution to the overall aerodynamic performance of the vehicle comes from the wing airfoil and therefore its careful design is paramount. In the case of the fast growing market of unmanned aerial vehicle (UAV) applications, the need for cost reduction and the miniaturization of the sensors payload is driving the designs to smaller scale and lower airspeeds. This brings the low Reynolds $(60,000<R e<500,000)$ airfoil aerodynamic problem, where the boundary layer laminar separation bubble and consequent transition influences decisively the drag coefficient [1]. Most formal approaches in the design of airfoils try to change the airfoil geometry in order to directly minimize the drag coefficient for a given flight condition or angle of attack range or to match a given pressure distribution known to be favourable for a given application. Many researchers have concentrated their efforts on optimizing the turbulent boundary layer pressure recovery strategy to maximize the value and
extension of the low pressure region in the upper surface or to delay the transition further aft thus obtaining higher lift coefficient and/or smaller drag due to extensive laminar flow at moderate/high Reynolds number [2-4]. These concepts lead to laminar separation in airfoils designed in such a way at low Reynolds numbers due to laminar bubbles that protrude massively from the airfoil contour before the transition is triggered and the flow reattaches with a high local velocity, $U_{e}$, drop and loss of momentum, leading to an unbearable drag coefficient increase taking place. The concept of surface transition ramp is a result of the later airfoil design philosophy to low Reynolds number when the presence of a strategically placed laminar boundary layer adverse pressure gradient ramp in the airfoil pressure distribution at a given angle of attack can result in a condition of near minimum drag coefficient increase due laminar separation and detached boundary layer transition. The idea is that when the separation does occur, the transition and reattachment follows shortly after. Selig [5] pioneered the application of the work previously done by Eppler [6] where, based in an inverse design by conformal mapping to reach a prescribed inviscid pressure distribution, where segments of constant velocity along the airfoil velocity distribution are prescribed for given angles of attack. Each discrete segment turns into a transition ramp when the angle of attack increases a certain amount above the value prescribed for constant velocity. So, this prescription of angles of attack along the contour, where the higher the prescribed angle of attack the closer the constant velocity segment is from the near leading edge stagnation point in the upper surface and the opposite for the airfoil's lower surface, correlates strongly with the curve of laminar separation/transition position (non-dimensional position along airfoil chord, $X t r / c$ ) versus angle of attack/lift coefficient (the transition curve). In practice, one can realize the correlation of these transition curves with the airfoil's drag polar (see Fig. 1 showing drag polars and transition curves for the Selig's SG604x airfoil series).

One problem in the inviscid inverse design formulation in PROFOIL, as devised by Selig, to manipulate the transition curves [7] is that it does not constitute a direct control of the transition curve position for a given Reynolds number operating condition since the inviscid velocity distribution is not the actual velocity distribution because of the real flow viscosity: it would need some iteration to arrive at a desired objective transition curve. Another issue is that rather than defining the airfoil's velocity distribution with a constant velocity segment it would make more sense to define it with a constant velocity position such that the velocity distribution can be defined in a continuous way rather than in a discrete way.

One can observe in Fig. 1, and in general, that for a given lift coefficient, as long as no significant turbulent separation takes place, the further aft the separation/transition occurs, the smaller the drag coefficient. The desired objective for good airfoil performance seems to be delaying as further aft as possible the transition position for a given design Reynolds number without incurring in significant turbulent separation. So, this furthest aft transition position is imposed by the turbulent pressure recovery method. The Stratford turbulent recovery [8] is the choice that would allow the most aft transition to be implemented at a given angle of attack or lift coefficient. For the same position of transition, in the airfoil's upper and lower surfaces, the drag coefficient will be smaller if the transition curve has a shallower slope along Xtr. The later observation allows the SG6041 airfoil to have peak efficiency at a smaller lift coefficient than the others. But the consequence of using a shallow slope to increase the efficiency in a given design lift coefficient is that the turbulent pressure recovery strategy gets compromised around that design condition when pursuing a high maximum lift coefficients.


Figure 1: Performance and transition curves predictions of Selig's SG604x series airfoils using XFOIL [5]

Having this in mind, it is observed that the steepest transition curve could produce the highest lift airfoil for a given fixed wing aircraft application (with constant $\operatorname{Re} \sqrt{C_{l}}$ ). An example application is to further improve the performance of the airfoil most used in high payload fractions design-build-fly competitions, the Selig S1223 airfoil, while using an optimization algorithm coupled to a viscous/inviscid formulation to pursue the desired transition curves.

## 2 AIRFOIL OPTIMIZATION

Formal numerical airfoil optimization has received increased attention from the scientific and engineering community because its performance is of utmost importance to the overall efficiency of aircraft. Both aerodynamic analysis tools and optimization algorithms have been used to optimize airfoils for specific applications, being the most common the high subsonic speed commercial transport [9-12]. The most utilized optimization algorithms applied to airfoil design range from gradient-based [9,12,13] to stochastic algorithms [10] and adjointmethods. Some optimization work has also been produced for low $R e$ applications [13,14], particularly in the design of UAV airfoils, where gradient-based algorithms are adopted even though the preferred approach still appears to be the inverse design method [15] rather than numerical aerodynamic shape optimization.

Several steps are required in order to solve an airfoil shape gradient based optimization problem: the airfoil must be mathematically defined in such a way that it is possible to change its shape; a method must be implemented to account for the deformation of the airfoil; an aerodynamic solver must be selected to obtain the necessary values to compute the objective
function and constraints; a method must be chosen to compute the gradients, and finally; an optimization algorithm must be used. In the following paragraphs, the different methods selected for each step are described.

An in-house low speed airfoil optimization code [13,16], designed for aerodynamic shape optimization of airfoils subject to operational and geometric constraints, is modified and used in this work.

### 2.1 Airfoil Geometry Parameterization

Two uniform cubic B-splines are used to discretize the airfoil: one for the thickness distribution and one for the camber line. The airfoil section is obtained by combining the camber line and the thickness. The coordinates of the points on the surface of the airfoil are obtained from the following expressions for all longitudinal $x$ coordinates

$$
\left\{\begin{array}{l}
z_{u}(x)=z_{c}(x)+z_{t h}(x)  \tag{1}\\
z_{l}(x)=z_{c}(x)-z_{t h}(x)
\end{array}\right.
$$

where $\left(x, z_{u}\right)$ and $\left(x, z_{l}\right)$ are points on the upper and lower surfaces, respectively and $z_{t h}$ and $z_{c}$ are the abscissas of the thickness distribution and the camber line, respectively, for the given $x$ ordinate.

In this optimization the vertical location ( $z$-coordinate) of the control points of the uniform cubic B-splines are used as the design variables. The airfoils are represented using one Bspline with 8 control points for the thickness distribution and another B-spline with 7 control points for the camber line. From the total of 15 control points, 12 are used as design variables. In particular, the control points numbered from 1 to 7 are used in the thickness distribution and those from 10 to 14 are used in the camber line, as shown in Fig. 2(a), are used as design variables. The two control points, representing the thickness distribution, aligned at the $x=0$, one at the fixed point $(0,0)$ and the other placed in the positive $z$ direction, are used to force the different airfoils to have the same leading edge point. Furthermore, the $z$ position of the moving control point (point 7) at the leading edge is also used as a design variable. This variable is used to control the sharpness of the leading edge during optimization.

In Fig. 2(b) the B-splines are used to represent the Selig S1223 airfoil [1c]. It can be observed that the two B-spline representation accurately defines the Selig S1223 airfoil shape. The leading edge and trailing edge areas are the regions that show some deviation from the original geometry due to the large curvature of the airfoil surface in the region and the small number of spline control points used. The distribution of the B-spline control points along the airfoil chord is chosen in such a way as to give a good representation of the airfoil geometry.

In most cases involving thick airfoils, a denser panelling is used near the leading and trailing edges, where the radius of curvature is smaller and/or the rate of change of the flow state variables is higher. A frequently used method for dividing the chord into panels with larger density near the edges is the full cosine method. In this method, a half-circle is divided into equally spaced angles, $\Delta \beta$, as shown in Fig. 3, and the $x$ coordinate is obtained from

$$
\begin{equation*}
x=\frac{c}{2}(1-\cos \beta) \tag{2}
\end{equation*}
$$

If $n$ chordwise panels are needed, then $\Delta \beta=\pi / n$ and the angle for the panel corner points $x_{i}$
is given by

$$
\begin{equation*}
\beta_{i}=(i-1) \Delta \beta \quad \text { for } \quad i=1, n+1 \tag{3}
\end{equation*}
$$



Figure 2: (a) Control points representing the B-splines used for camber line and thickness distribution of the airfoils and corresponding airfoil geometry; (b) Comparison of Selig S1223 airfoil and its B-spline representation


Figure 3: Airfoil surface panel distribution

Given the number of panels required for the airfoil surface, the panel distribution is obtained from Eqs. (2) and (3). Then, knowing the B-spline representations of both the thickness distribution and camber line by having their control points, Eq. (1) can be used to calculate the panel corner points for the upper and lower surfaces of the airfoil.

### 2.2 Aerodynamic Analysis

The 2-dimensional (2D) aerodynamic coefficients and aerodynamic properties of the airfoil as functions of angle of attack (AOA) and Reynolds number (Re) are obtained using the solver of the XFOIL code [17]. In XFOIL, the steady Euler equations in integral form are used to represent the inviscid flow, and a compressible lag-dissipation integral method is used to represent the boundary layers and wake. The entire viscous solution (boundary layers and wake) is strongly interacted with the incompressible potential flow via the surface transpiration model which permits proper calculation of limited separation regions. Results from XFOIL have been compared against experimental data with good agreement [16].

### 2.3 Optimization Approach

The general optimization problem can be stated as

$$
\begin{array}{cc}
\text { minimize: } & f(v) \\
\text { subject to: } & h(v)=0  \tag{5}\\
g(v) \geq 0
\end{array}
$$

where the design variables, $v$, may be flight and/or geometric parameters and the equality, $h(v)$, and inequality, $g(v)$, constraints may be lift coefficient and/or geometric parameters, for example.

The aerodynamic shape optimization is carried out with the sequential quadratic programming (SQP) constrained optimization algorithm of FFSQP3.7 [18]. The purpose of the FFSQP3.7 algorithm is the minimization of an (in general nonlinear) differentiable real function subject to (in general nonlinear) inequality and equality constraints. Numerical techniques, such as FFSQP3.7, generally assume that the design space is convex, continuous, and unimodal. Because of this, numerical techniques tend to converge quickly to a local optimum close to the initial design point. Thus, the effectiveness in finding a global optimum is highly dependent on the topology of the design space and the choice of the initial design point. Nonetheless, SQP has been shown to produce good results [19].

The gradients of the objective function and constraints are a requirement of any gradientbased optimization algorithm. In this work, the gradients are computed using forward finitedifferences, which enables the problem of finding the gradients to be treated as a black box. Therefore it can be used with any fluid flow solver because it does not involve changes in the solver's code.

### 3.4 Aerodynamic Shape Optimization

The objective of the airfoil design is to minimize a cost function that produces a good or a set of good airfoil characteristics. In order to achieve this, a tool that searches for the best airfoil geometry is used, which may take into account geometric constraints or performance constraints imposed by the user. Figure 4 shows a flow chart that illustrates the implementation of the aerodynamic shape optimization tool. The code can be summarized as follows:

1. Create the airfoil using the B-spline approach;
2. Compute objective function, $f(v)$, and constraints, $h(v)$ and $g(v)$, of the optimization problem using the aerodynamic solver XFOIL;
3. Compute gradients of objective function and constraints using forward-differences;
4. Solve the optimization problem using the SQP method;
5. If the optimization problem has converged stop; if the optimization has not yet converged continue;
6. Use the new design variables to create new airfoil geometry and go to step 2 .


Figure 4: Flow chart of the airfoil aerodynamic shape optimization design tool

## 3 AIRFOIL DESIGN OPTIZATION CASE

Design-build-fly competitions have become popular within aerospace sciences students. Usually the design goal is maximum payload and/or endurance with some constraining requirements. In this scenario as in general small UAV applications high maximum lift coefficient, $C_{l} / C_{d}$ and $C_{l}^{3 / 2} / C_{d}$ values are a significant part of the airfoil design goal along with large relative thickness. From the authors experience one airfoil seems to be the most widely used: Selig's S1223 [4]. An effort to improve this airfoil according to extensive laminar flow region and steep transition curve design philosophy was thus pursued by setting the desired transition curves by the transition positions in the upper surface and lower surface in $5 C_{l}$ values within the useful operation envelope of the initial S1223 airfoil for a $R e \sqrt{C_{l}}=200,000$, which is a representative value for a typical design-build-fly application (see Fig. 6).

### 3.1 Problem Definition

Finding the airfoil geometry that gives the desired transition curves, which are essentially the transition position, Xtr, as a function of $C_{l}$, on the upper and lower surfaces of the airfoil, is implemented by minimizing the square difference between the desired curves and those produced by the current airfoil geometry.

The optimization problem statement for the study is written as

$$
\left.\left.\begin{array}{rl}
\operatorname{minimize:~} f(v)=w \sum_{j=1}^{n}\left(X t r_{u, j}-X t r o b j_{u, j}\right)^{2}+(1-w) \sum_{j=1}^{n}\left(X t r_{l, j}-X_{t r o b j}^{l, j}\right.
\end{array}\right)^{2}\right)
$$

where $\mathrm{Xtr}_{j}$ is the obtained transition position and $\mathrm{Xtrob}_{j}$ the objective transition position corresponding to $C_{l, j}$. The indices $u$ and $l$ indicate the upper and lower airfoil surfaces, respectively, and the index $j$ denotes the $j$ th analysis $C_{l}$ point. The parameter $w$ is a weighing factor which, in this study, is taken as 0.5 to give the same importance to the upper surface and lower surface transition curves.

The use of a constant value of $\operatorname{Re} \sqrt{C_{l}}$ is representative of a set of flight conditions where the lift coefficient is adjusted as speed varies so that total lift is maintained unchanged. In this study a total of five ( $n=5$ ) lift coefficient values ranging from 1.1 to 2.1 are chosen to be representative of the flight envelope required for the airfoil to be designed. From Eq. (7) it follows that the $R e$ for these $C_{l}$ values range from 138,000 to 190,700 . A minimum relative thickness of $1 \%$ at the trailing edge, $(t / c)_{T E}$, is set to avoid too thin a trailing edge which is difficult to build and prone to breakage during ground handling.

The initial airfoil selected is the S1223 which is known to have good performance for the given application and is widely used in high lift radio controlled aircraft wings with low speed, heavy payload requirements. The airfoil representation using the two B-spline approach described above is shown in Fig. 5.

### 3.2 Results

The resulting airfoil from the optimization is shown in Fig. 5 along with the initial S1223 in the two B-spline representation. The final maximum relative thickness is $11.71 \%$ compared with the initial $12.04 \%$, a negligible change for practical applications but the corresponding position is significantly shifted aft from $20.6 \%$ to $24.5 \%$ of the chord. This results in an important improvement for the main spar position which can be placed closer to the center of pressure and leaving more room in case a D leading edge structural configuration is used. The maximum camber position decreased from $8.72 \%$ to $8.49 \%$ and the corresponding position was displaced slightly back from the initial $48.4 \%$ to $50.0 \%$.

The final airfoil transition curves, lift curve and drag polar are displayed in Fig. 6 with solid lines. In the same figure, the curves from the initial airfoil are drawn with dashed lines. The objective transition points versus lift coefficient that defined the objective transition curves are also shown. It is seen that the maximum lift coefficient of 2.18 from the initial airfoil S1223 is not reached. The maximum final airfoil lift coefficient is 2.12 , a difference smaller than $3 \%$ although the stall seems more abrupt. The reason can be related to the difference between the objective transition point of the upper surface and the actual transition point reached in the final airfoil design. With the current algorithm a transition curve cannot be described on the upper surface for positive $\mathrm{dXtr} / \mathrm{dC} C_{l}$ values while convergence between the
final airfoil transition curve in the upper surface and the objective is weak even at slightly negative $\mathrm{d} X t r / \mathrm{dC}_{l}$ near the maximum lift coefficient. This can be explained by the large sensitivity of the $C_{l}$ curve to small perturbations in the leading edge geometry and the limited number of design parameters at the leading edge prevent the required geometric resolution from being achieved. On the other hand, the final airfoil drag polar shows a significant improvement below a lift coefficient of 1.75 .


Figure 5: Airfoil optimization results for $\operatorname{Re} \sqrt{C_{l}}=2 \times 10^{5}$ : initial airfoil (Selig S1223 B-spline representation) and final airfoil geometries


Figure 6: Airfoil optimization results for $\operatorname{Re} \sqrt{C_{l}}=2 \times 10^{5}$ : drag polar, lift and pitching moment coefficients and transition curves

The improvements in performance can be observed in Fig. 7. The final airfoil aerodynamic efficiency, $C_{l} / C_{d}$, is significantly higher and it extends over a wider $C_{l}$ envelope. The maximum value of $C_{l}^{3 / 2} / C_{d}$ is not improved but the range of high values is extended to much lower lift coefficients. This is beneficial in actual flight because it is important to have a good margin below the maximum lift coefficient to prevent an unintentional stall.


Figure 7: Airfoil optimization results for $\operatorname{Re} \sqrt{C_{l}}=2 \times 10^{5}$ : lift-to-drag ratio and lift ${ }^{3 / 2}$-to-drag ratio


Figure 8: Airfoil results for $\operatorname{Re} \sqrt{C_{l}}=1.5 \times 10^{5}$ : drag polar, lift and pitching moment coefficients and transition curves

In order to assess the performance of the optimized airfoil at a lower Reynolds number, aerodynamic curves were obtained with XFOIL for $\operatorname{Re} \sqrt{C_{l}}=1.5 \times 10^{5}$. Figures 8 and 9 show the results obtained. These clearly indicate that the new design is overall superior to the initial airfoil. One particular aspect is that the stall behavior is smooth, an important requirement for good flight handling qualities at low speed.


Figure 9: Airfoil results for $\operatorname{Re} \sqrt{C_{l}}=1.5 \times 10^{5}$ : lift-to-drag ratio and lift ${ }^{3 / 2}$-to-drag ratio

## 4 CONCLUSIONS

- Aerodynamic shape optimization using a gradient-based algorithm was performed to design a low Reynolds airfoil to match a set of transition curves. The method implemented produced a good performing airfoil but revealed some convergence difficulties near the higher lift coefficients, which were attributed to the airfoil parameterization in the leading edge region.
- The approach of using the transition curves as objective functions proved useful in producing a good airfoil design.
- This work is a preliminary investigation on this type of airfoil design approach and requires further improvements on the optimization algorithm as well as on the airfoil parameterization scheme.


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