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STRUCTURAL OPTIMISATION AS A MOVING BOUNDARY PROBLEM USING LEVEL SET FUNCTIONS

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Abstract. This extended abstract presents topology optimisation which uses level set functions representing the moving boundaries. The level set function based approach to topology optimisation has gained much popularity in the recent years due to its numerical stabilities and clear boundary representation of the solution. One advantage of the level set representation is its inherent capability to handle topological changes such as merging and splitting boundaries. We have developed a stable hole nucleation algorithm which makes the level set formulation completely suitable for topology optimisation. We demonstrate that our level set topology optimisation, both in 2D and 3D, have good convergence properties and less dependency on the initial design. We apply this to typical structural optimisation problems as well as coupled aero-structural problems. As coupled multidisciplinary optimisation problems have multiple optima, we find that the solutions 3D level set topology optimisation produce can be quite different from the solutions from the previous element-based approaches and simplified 2D solutions, suggesting potential alternatives.

1 INTRODUCTION

The level set method is a boundary or an interface tracking method. It was first introduced by Osher and Sethian [1] and since then it has been applied a range of areas such as image processing and multiphase flows. In the field of structural optimisation, the level set method can be used to track the curves or surfaces that define structural features as they are optimised over iterations. This following sections describe the level set topology optimisation method with a stable hole creation algorithm which will be demonstrated using numerical examples.

2 TOPOLOGY OPTIMISATION

Topology optimisation is the most general form of structural optimisation; of all structural optimisation, topology optimisation finds an optimal solution that is least dependent on the initial design. A common approach to topology optimisation is to formulate the problem as a material distribution problem where the available design domain is discretised with finite elements. Optimisation then determines whether each element should or should not exist iteratively. This formulation makes the optimisation problem a large-scale binary problem which is typically relaxed to a continuous problem with design variables bounded between 0 and 1. This enables a gradient-based optimiser to solve the problem efficiently however the solutions with design variables between 0 and 1 does not usually represent a physical and manufacturable structure as this means a structure with material properties continuous varying throughout the structure. Therefore, the solutions with non-0/1 variables are penalised. This approach has been applied to many disciplinary problems and demonstrated to work well but it is well known that a complete elimination of non-0/1 solutions can be difficult to obtain and the numerical procedure introduces various parameters to which the solutions and convergence can be highly sensitive.

An alternative approach to topology optimisation using the level set method was introduced relatively recently, [2]. Since then, there has been a flurry of activities maturing this approach. One attractive advantage is that the level set method obtains clear boundaries defining the general layout of the optimising structure at every iteration and eliminates the non-0/1 solutions completely. We will first outline our level set based topology optimisation method with a hole creation algorithm. The following sections will then show the example results to demonstrate that our method eliminates chequerboarding, a commonly known numerical instability in topology optimisation and reduced dependency on the initial solution [3]. The last example shows the application to a coupled multidisciplinary problem, aero-structural topology optimisation of an aircraft wing.

2.1 Level Set Topology Optimisation Method

The level set method defines the structural boundaries to be where the level set ϕ, is zero, (1).

$$\begin{cases} \phi(x) > 0, & x \in \Omega_S \\ \phi(x) = 0, & x \in \Gamma_S \\ \phi(x) < 0, & x \notin \Omega_S \end{cases} \tag{1}$$

where Ω_S is the domain of the structure and Γ_S is the boundary of the structure. The compliance of the structure, $C(u, \phi)$ is minimized subject to an upper limit on structural volume:

$$\begin{aligned} &\text{Minimize}: C(u,\phi) = \int_\Omega E\varepsilon(u)\varepsilon(u)H(\phi)d\Omega \\ &\text{Subject to}: \int_\Omega H(\phi)d\Omega \le Vol^* \\ &\qquad \int_\Omega E\varepsilon(u)\varepsilon(v)H(\phi)d\Omega = \int_\Omega bvH(\phi)d\Omega + \int_{\Gamma_S} fvd\Gamma_S \\ &\qquad u|_{\Gamma_D} = 0 \quad \forall v \in U \end{aligned} \tag{2}$$

where Ω is a domain larger than Ω_S such that $\Omega_S \subset \Omega$, Vol^* is the limit on material volume, E is the material property tensor, $\varepsilon(u)$ the strain tensor under displacement field u, U is the space of kinematically permissible displacement fields, v is any permissible displacement field, b are body forces, f are surface tractions and $H(\phi)$ is the Heaviside function.

When applied to topology optimisation, the level set method incorporates shape sensitivity in computing the velocity function of a typical Hamilton-Jacobi equation, (3).

$$\phi_i^{k+1} = \phi_i^k - \Delta t \left|\nabla \phi_i^k\right| V_{n,i} \tag{3}$$

where $V_{n,i}$ is a discrete value of the velocity function acting normal to the boundary at point i, Δt is a discrete time step and k is the current iteration. In the case of (2), the velocity function also includes the Lagrangian multiplier, λ for the volume constraint, thus giving (4).

$$V_n = \lambda - E\varepsilon(u)\varepsilon(u) \tag{4}$$

While this *primary* level set function modifies, merges and splits existing boundaries, it is not possible to create a new boundary, i.e. a hole. We do this by introducing a *secondary* implicit level set function, $\bar{\phi}(x)$ [4]. It can be conceptually explained as the additional third dimension in the context of two-dimensional design domain, i.e. fictitious thickness. The secondary implicit level set function is initialized to an artificial thickness, $\bar{h}$, (5).

$$\bar{\phi}^0(x) = \begin{cases} +\bar{h}, & x \in \Omega_S \\ -\bar{h}, & x \notin \Omega_S \end{cases} \tag{5}$$

The secondary level set function is updated along with the primary level set function using (6).

$$\bar{\phi}_i^{k+1} = \bar{\phi}_i^k - \Delta t \bar{V}_{n,i} \tag{6}$$

A new hole is then created when $\bar{\phi}(x)$ becomes negative within the region of Ω_S and the new hole is added to the primary level set function by simply copying $\bar{\phi}(x)$ onto $\phi(x)$ within Ω_S. This inherent link between the primary and secondary level set functions forms a meaningful link between shape and topological optimisation, determining when and where to create a hole consistently.

2.2 A 2D Beam with Three Load Cases

We apply the level set topology optimisation with hole creation of Section 2.1 to a beam with three load cases shown in figure 1(a). Each load case has a magnitude of 2.0 and a weight of 1.0. The material properties are 1.0 and 0.3 for Young's modulus and Poisson's ratio, respectively. The beam is discretized using 200 × 50 unit sized square elements and the volume constraint is set to 40% of the design domain. Starting from the fully populated domain, the structure is optimised through figures 1(b)-(d), where figure 1(d) depicts the optimum solution coverged after 144 iterations with total compliance value of 4.67×10^2. The convergence history for this example is shown in figure 2. It is clear that the level set method creates smooth and well-defined boundaries throughout optimisation and holes emerge as required. The hole creation does not cause a sudden discontinuity in the convergence history of figure 2, indicating that when and where the holes are created, merged and split are optimal. It is also noted that there are no chequerboarding and this numerical stability is consistent in our experience.

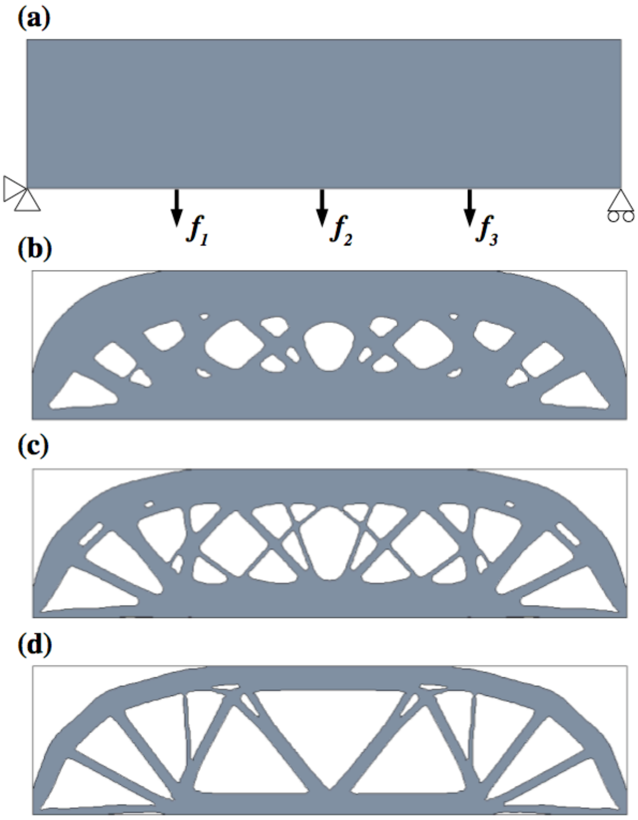


Figure 1: Beam optimisation for multiple load cases: (a) initial design; (b) 25 iterations; (c) 40 iterations; (d) solution after 144 iterations

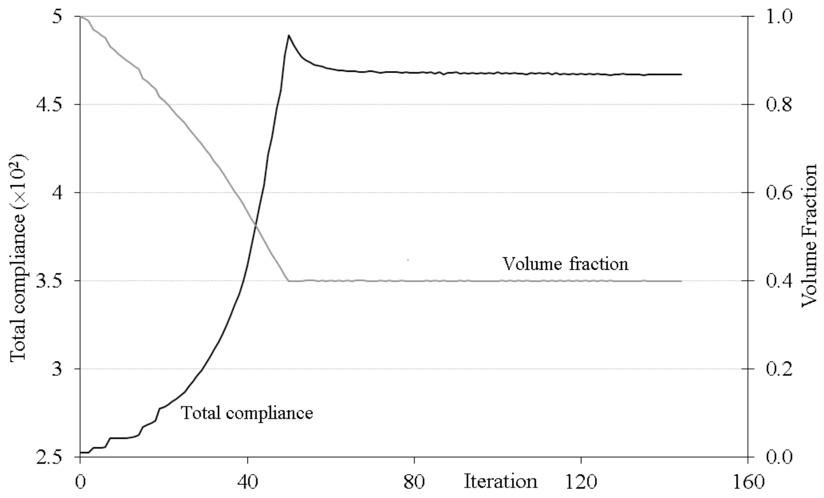


Figure 2: Convergence history of the beam optimisation

2.3 A 3D Cantilever Beam

This demonstrative example of a 3D cantilever beam is optimised for two load cases, one vertical and one horizontal loads at the centre. The other end is clamped. The beam is 45units long and the maximum cross-section is 20 × 20 unit^2. The volume constraint is set at 25% of the design domain.

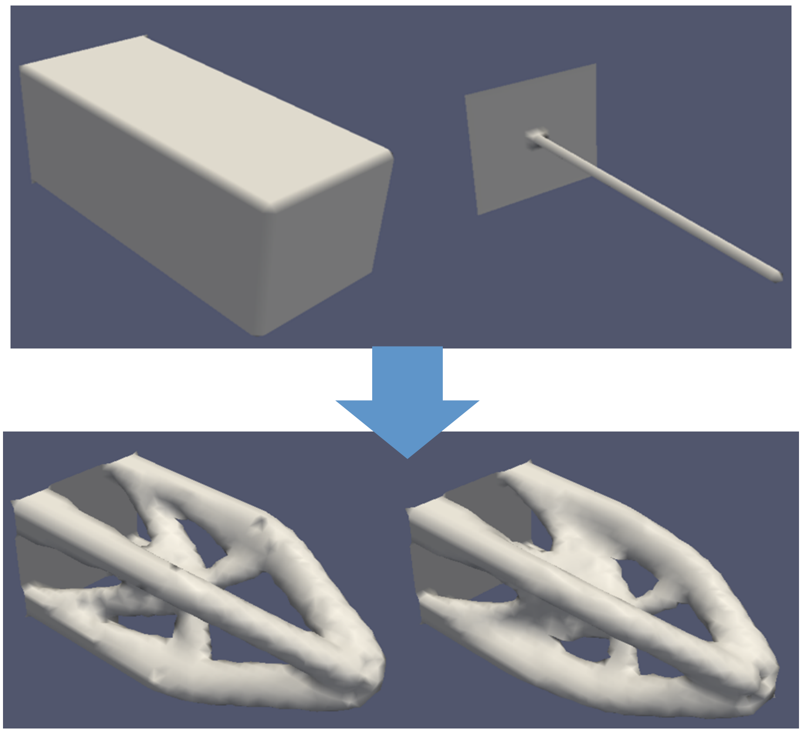


Figure 3: 3D canilever beam optimisation with two initial solutions.

We optimise this problem twice, the first time starting with the fully populated design domain and the second with the minimum structure linking the boundary and loading conditions by a thin beam, figure 3. We observe that the optimum solutions of the both runs agree favourably, with less than 0.5% difference in the compliance values between the two solutions. This shows that our level set method is robust and has reduced sensitivities to the starting solution.

2.4 Aero-Structural Wing Optimisation

We perform a preliminary study of 3D optimisation of the internal wing structure with full fluid-structure interaction used to update the aerodynamic loading during optimisation. The aerodynamic loading on the wing is calculated using the Double Lattice Method. The topology optimisation procedure is applied to the internal structure of a simple linearly tapered unswept wing box model with a 51×20×7 regular fixed finite element mesh. The wing is clamped at the root under a cruise condition. The top and bottom skins are fixed and excluded from optimisation. The volume constraint is set at 35%.

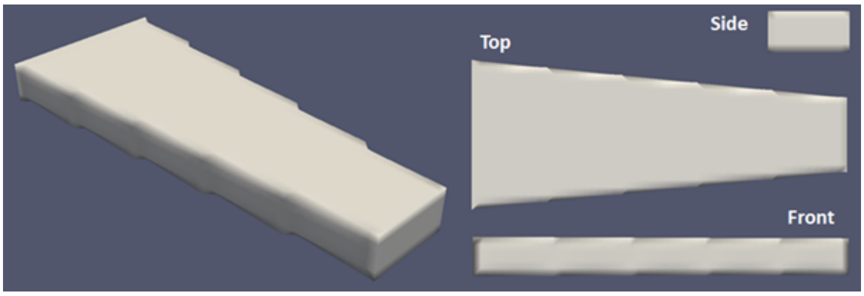
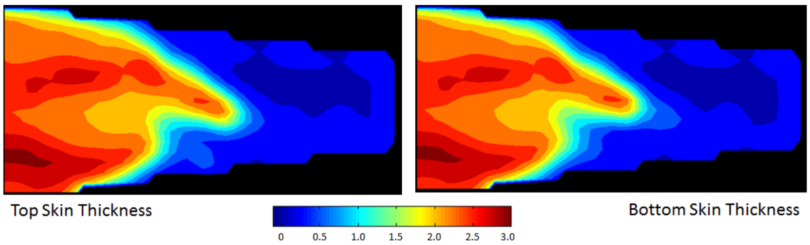


Figure 4: Geometry of a tapered unswept wing box model

(a)



(b) (c)

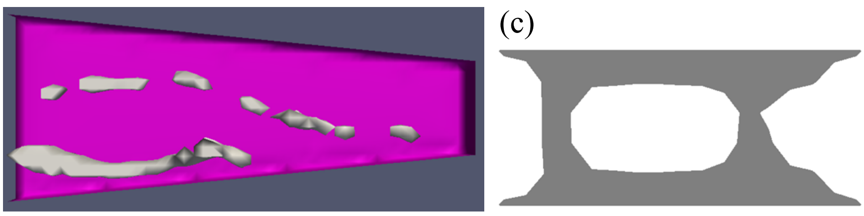


Figure 5: Optimum solution, (a) top and bottom skin thickness distribution; (b) internal column distribution; (c) cross-sectional view at span position 18

Figure 5 shows the optimum solution of the wing box. Looking at figure 5(a), the skin distribution along the top and bottom are nearly identical, with the root having the maximum thickness and gradually decreasing towards the tip. Perhaps what is the most distinctive about the optimum solution is the column like stiffeners connecting the top and the bottom skins, shown in figure 5(b). Near the root, the configuration looks somewhat reminiscent of two spar arrangements, reducing to a single "spar" arrangement towards the tip. These spar-like stiffeners are somewhat continuous near the room where the bending loads are the most significant, then they become discrete columns as the loads are reduced towards the tip. The other interesting feature to note is the skin thickness. It is significantly greatly than the typical skin thickness of the conventional wing configuration and figure 5(c) suggests a configuration similar to an I-beam. This is an intuitive characteristic as the wing is predominantly under bending during cruise. This preliminary optimisation result shows that there may be alternative configuration that may be more optimum than the conventional configuration and topology optimisation can be used to explore the potentially revolutionary optimum designs.

4 CONCLUSIONS

This extended abstract described the level set topology optimisation method and the new hole creation algorithm. Using this method, a few demonstrative examples are shown both in 2D and 3D: the chequerboarding is naturally eliminated and the method is not strongly dependent on the initial solution. Multidisciplinary topology optimisation was applied to a simple aircraft wing box under coupled aero-structural considerations. We see that the resulting structure is far from the conventional wing configuration suggesting that there is a potential for significant weight savings via revolutionary design changes. This warrants further studies.

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