We present a technique to smooth and untangle triangular and quadrilateral meshes on parameterized surfaces. To this end, we first extend any distortion measure for planar elements to elements on parameterized surfaces. The resulting distortion depends on the parametric coordinates of the element vertices. Then, we extend a planar measure that has a known untangling technique (such as the shape distortion measure [1] and the technique presented in [2]). Thus, a mesh on a parameterized surface can be smoothed and untangled finding the parametric coordinates of the element vertices that minimize the extended total distortion. The presented technique ensures that element nodes are always on the original surface. In addition, the optimal location of the nodes is independent of the surface parameterization. That is, the technique can be used to directly optimize meshes on bad parameterized CAD surfaces.

**PARAMETRIC DEVIATION MEASURE FOR SURFACE MESHES**

The proposed objective function measures the inverse of the quality of a triangle that lies on a given parametric surface:

\[
\eta_3 : \mathbb{T}_0 \subset \mathbb{R}^2 \to \eta_3(u_0, u_1, u_2) \to \bar{T}(\bar{\varphi}(u_0, u_1, u_2)) \to \mathbb{R}
\]

\(\bar{T}\): embedding of a 3D triangle on the plane

\(\bar{\varphi}\): parameterization

\(\eta_3\): distortion measure for planar elements

Objective function: 

\[
K^p(u) = \left( \sum_{k=1}^{m} \left( \eta_3^k(u) \right)^p \right)^{1/p}, \quad k = 1, \ldots, m
\]

Untangling objective function based on the technique presented in [2]. Quadrilateral case generalized from the vertex triangles as shown in [1].

**Independence of the parameterization**

If the parameterization is \(C^1\), the result is independent of the parameterization:

- **Parameterization 1**
- **Parameterization 2**

**PROPERTIES**

- **Initial mesh**
- **Tangled mesh**
- **Smoothed and untangled mesh**

**EXAMPLE: LINKING ROD**

- **Initial mesh**
- **Smoothed mesh**

References

