Hybridizable Discontinuous Galerkin methods (HDG) [1] are a novel class of discontinuous Galerkin (DG) methods with very interesting characteristics. Similarly to static condensation in continuous Galerkin (CG), hybridization allows to reduce the globally coupled degrees of freedom to those of an approximation of the solution defined only on the boundaries of the elements. In addition, unlike all the others known DG methods and standard CG, HDG exhibits optimal convergence in $L_2$ norm not only for the primal unknown of the problem but also for its derivative, opening the path to an element-by-element post-process which provides a super-convergent solution. Here, HDG is applied to the solution of incompressible flows. Fractional-steps methods for HDG are presented for the solution of 3D problems. Then, a p-adaptive technique is introduced [2], allowing to adapt the polynomial degree of the approximation in each element. The adaptive technique is driven by an error estimator based on the super-convergent properties of HDG and involves only elemental computation.

**Abstract**

Problems involving moving reference frames: in fact, the complex flow structures cover, in different time instants, the whole computational mesh.

**References**


**HDG: main features**

The HDG method retains the advantages of DG methods (stability from numerical fluxes, local conservation…) reducing drastically the coupled DOF with a procedure similar to static condensation of the interior nodes in CG.

**Numerical fluxes**

**Coupled DOF in DG and CG**

**Static condensation of interior nodes**

**Superconvergent post-process**

The element-by-element post-process in HDG provides a super-convergent solution, adding a negligible overhead in the computational cost.

**P-adaptivity**

The discontinuous character of the solution in HDG opens the path to p-adaptive simulations, allowing to increase the polynomial interpolation only where more accuracy is needed. The super-convergent solution provides an inexpensive and accurate error estimation, that is used to drive the adaptive procedure.

**Simulation of wind turbines**

Adaptive simulations are particularly effective for transient problems involving moving reference frames: in fact, the complex flow structures cover, in different time instants, the whole computational mesh.

**Simulation of a NACA airfoil**

The p-adaptive algorithm greatly simplify the design of the initial computational mesh, reducing for example the need of highly distorted elements for capturing the boundary layer.

**Fractional step methods for HDG**

The uncoupling of the pressure and velocity allows to circumvent the difficulties caused by the saddle-point nature of the variational formulation of the Navier-Stokes equations, were the pressure variable acts as a Lagrange multiplier of the incompressibility constrain.

**Transition to turbulence in a 3D Taylor-Green vortex problem**

Iso-surfaces of the z-component of the vorticity.