Proper Generalised Decomposition for coupled acousto-magneto-mechanical problems with application to MRI scanners

Guillem Barroso

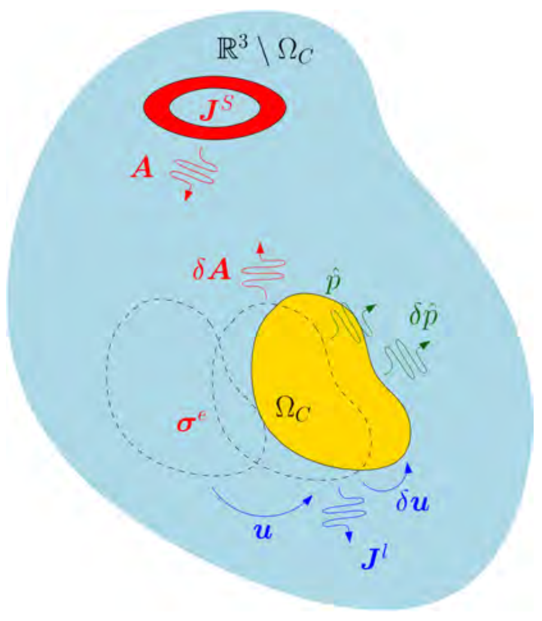
Academic & Industrial supervisors: Antonio J. Gil†, Paul D. Ledger†, Mike J. Mallett‡, Antonio Huerta*

† Zienkiewicz Centre for Computational Engineering, Swansea University, UK ‡ Siemens Magnet Technology, Oxford, UK * Universitat Politècnica de Catalunya, Barcelona, Spain

1. Introduction

Objective

- **Optimise design** process of **MRI scanners** by implementing a **reduced order method**

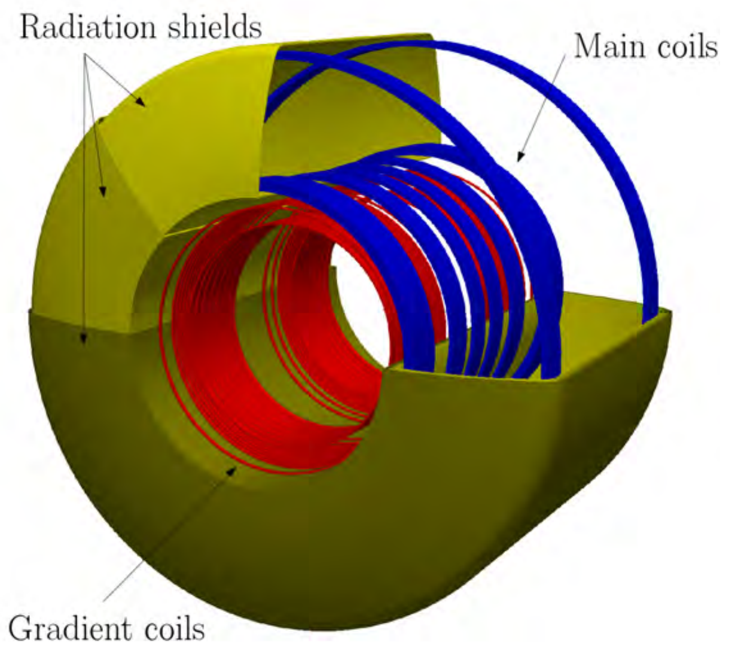


Challenges

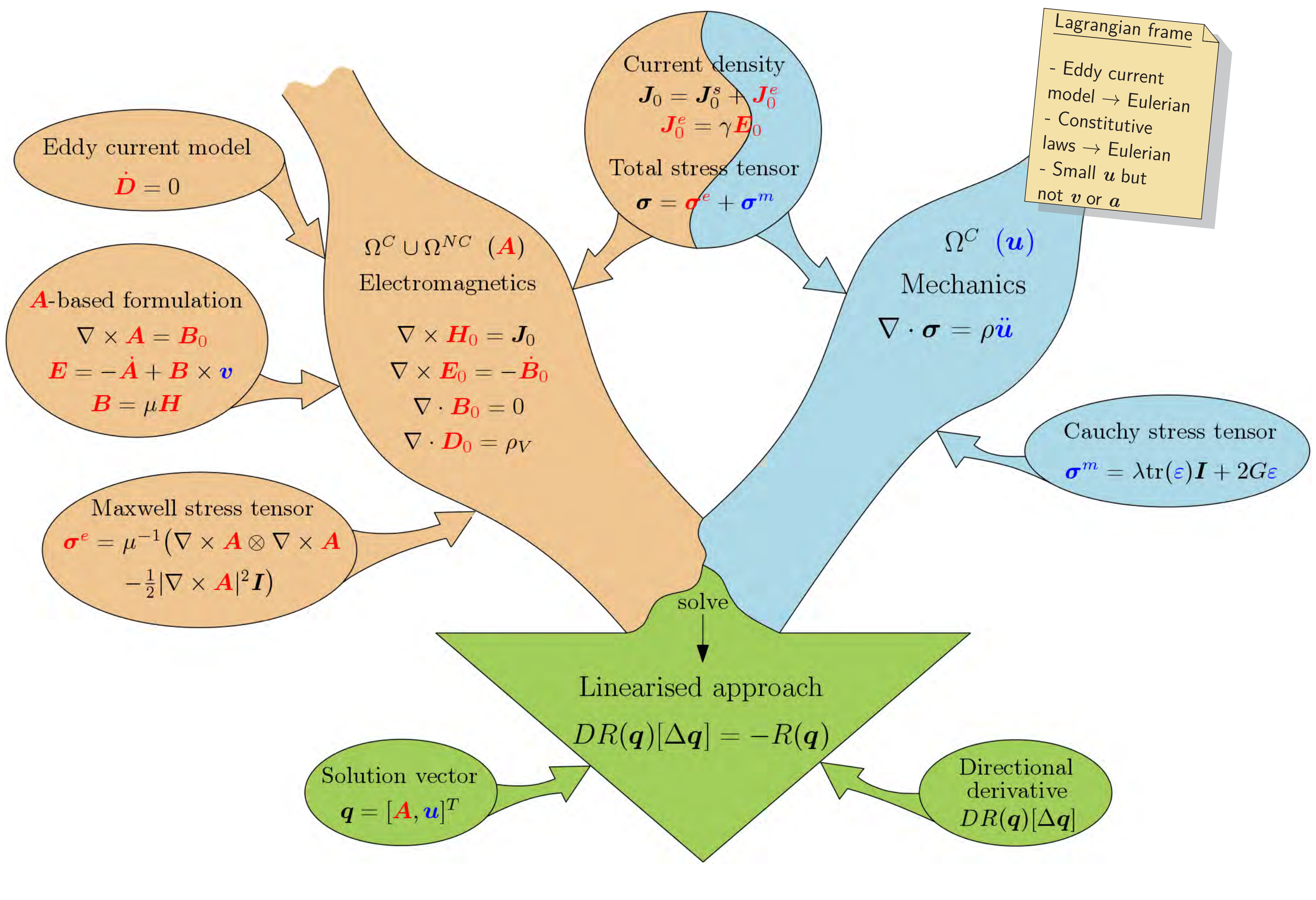
- Complex **coupled problem**
- Unwanted vibrations → **imaging artefacts** and **noise**
- **High computational cost** when designing coils

MRI basics

- Strong static (DC) magnetic field generated by the **main coils**
- **Gradient coils** emit transient (AC) magnetic field
- **Radiation shields** prevent electromagnetic radiation from escaping



2. Physical problem



Eddy current model
$\dot{D} = 0$

A-based formulation
$\nabla \times A = B_0$
$E = -\dot{A} + B \times v$
$B = \mu H$

Maxwell stress tensor
$\sigma^e = \mu^{-1}(\nabla \times A \otimes \nabla \times A - \frac{1}{2}|\nabla \times A|^2 I)$

Current density
$J_0 = J_0^s + J_0^e$
$J_0^e = \gamma E_0$
Total stress tensor
$\sigma = \sigma^e + \sigma^m$

$\Omega^C \cup \Omega^{NC}$ (A)
Electromagnetics
$\nabla \times H_0 = J_0$
$\nabla \times E_0 = -\dot{B}_0$
$\nabla \cdot B_0 = 0$
$\nabla \cdot D_0 = \rho_V$

Lagrangian frame
- Eddy current model → Eulerian
- Constitutive laws → Eulerian
- Small u but not v or a

Ω^C (u)
Mechanics
$\nabla \cdot \sigma = \rho \ddot{u}$

Cauchy stress tensor
$\sigma^m = \lambda \mathrm{tr}(\varepsilon) I + 2G\varepsilon$

solve

Linearised approach
$DR(q)[\Delta q] = -R(q)$

Solution vector
$q = [A, u]^T$

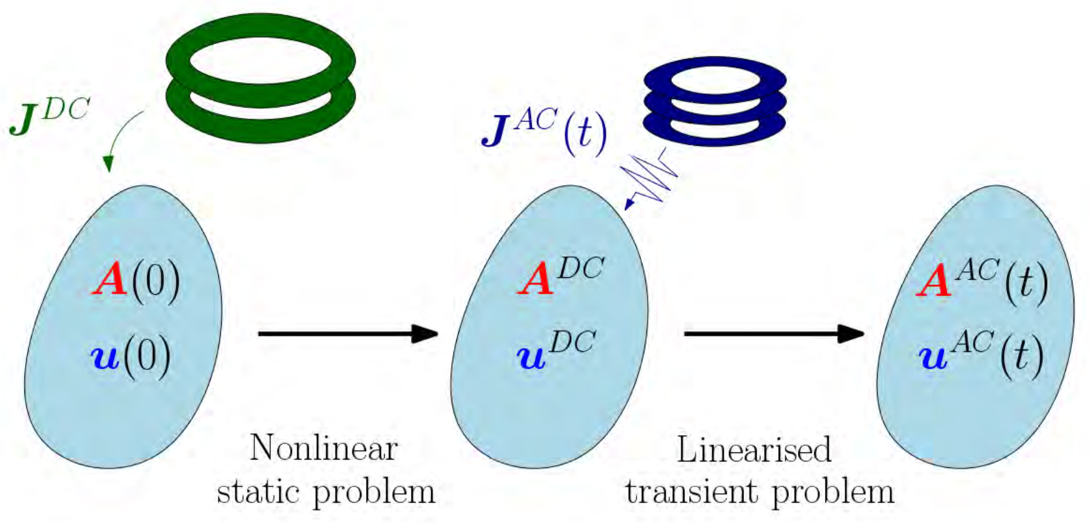
Directional derivative
$DR(q)[\Delta q]$

3. Numerical problem

Linearisation[1]

- Solve **static** DC field
- Solve **linearised** AC problem with DC field as initial state
- Recover **total solution** as

$$A(t) = A^{DC} + A^{AC}(t)$$
$$u(t) = u^{DC} + u^{AC}(t)$$



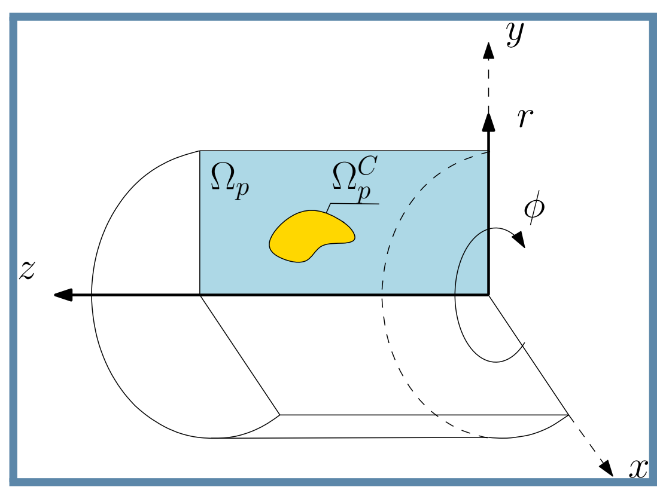
Axisymmetric formulation[1]

- Assume axisymmetry in **geometry** and **current source**

$$J^s(r, \phi, z) = J^s_\phi(r, z) e_\phi$$

- Use **cylindrical coordinates**

$$A(r, \phi, z) = A_\phi(r, z) e_\phi$$
$$u(r, \phi, z) = u_r(r, z) e_r + u_z(r, z) e_z$$



Frequency domain

- From now onwards **only dynamic AC problem** is **considered**
- Compute using **complex amplitudes** $A_\phi \rightarrow \overline{A}_\phi e^{i\omega t}$ and $u \rightarrow \overline{u} e^{i\omega t}$
- **Expensive frequency sweeps** when computing **integrated quantities** such as dissipated power $P^0_{\Omega^C}$ and kinetic energy $E^k_{\Omega^C}$

4. Proper Generalised Decomposition (PGD)

General d-dimensional PGD definition

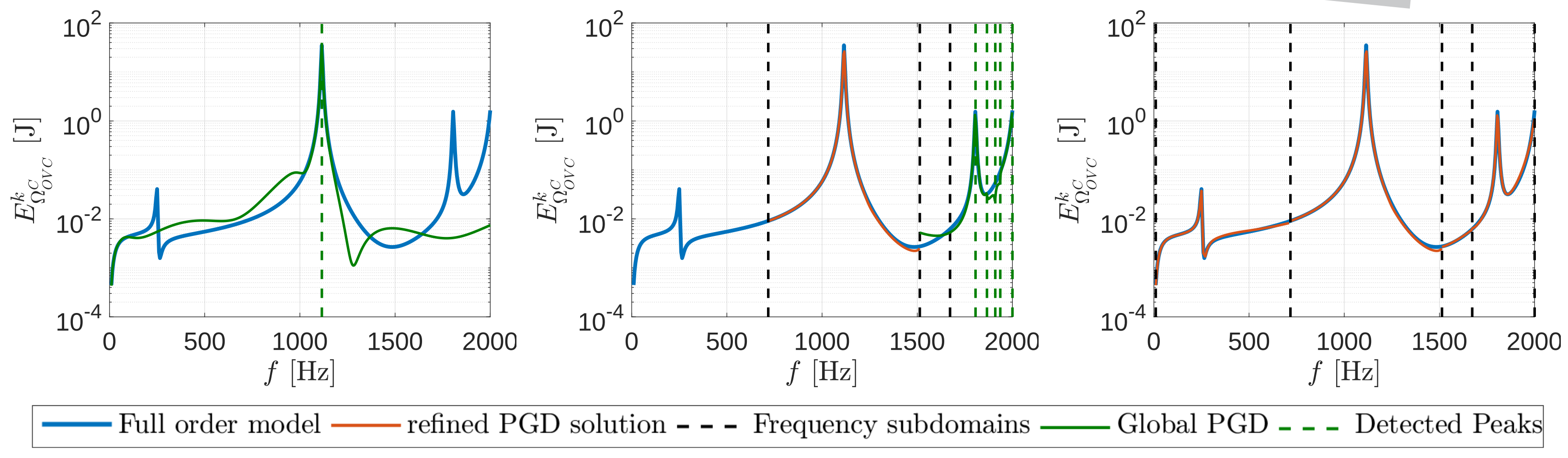
$$q^N(x_1, \ldots, x_d) = \sum_{n=1}^{N} \prod_{i=1}^{d} F_i^n(x_i)$$

- **Off-line stage**: obtain general solution $q^N(x_1, \ldots, x_d)$ by computing all $F_i^n(x_i)$
- **On-line stage** interpolate $q^N(x_1, \ldots, x_d)$ for a set of parameters $(x_1, \ldots, x_d)$

Regularised-adaptive PGD[2]

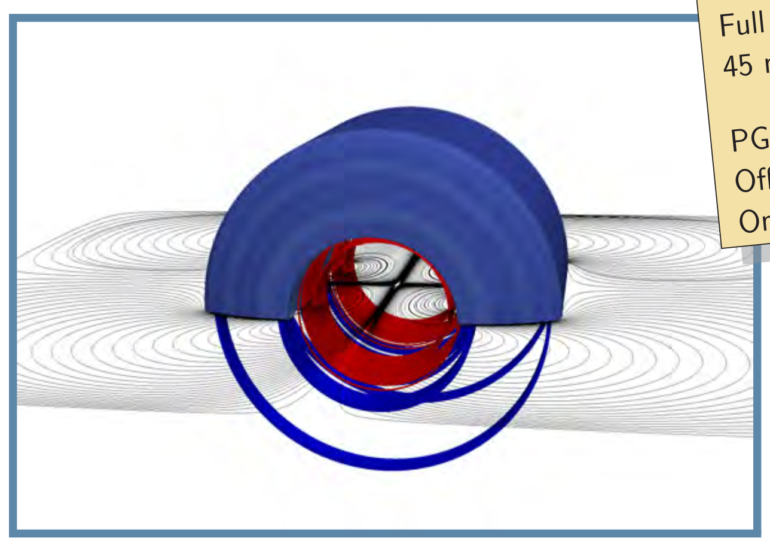
- **3D** parametric **problem** $q^N(r, z, \omega)$
- Conducting shell in a non-conducting domain
- Addition of **numerical regularisation** (Rayleigh coeff. α)
- Automatic **adaptive PGD splitting**

Splitting PGD:
1. Global PGD
2. Detect singularities
3. Divide f-domain
4. Refined PGD
5. Back to 1



5. Results

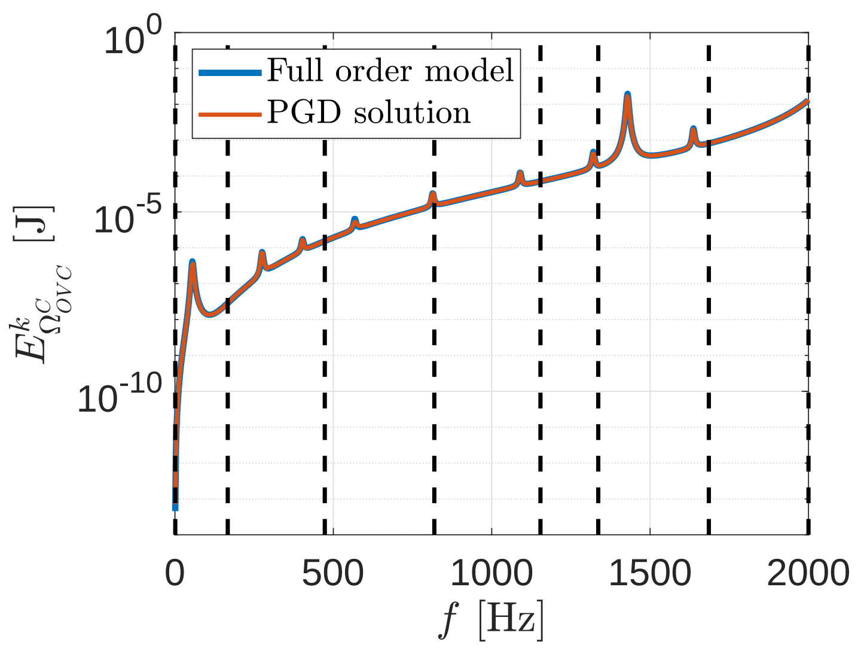
3D parametric full magnet problem $q^N(r, z, \omega)$



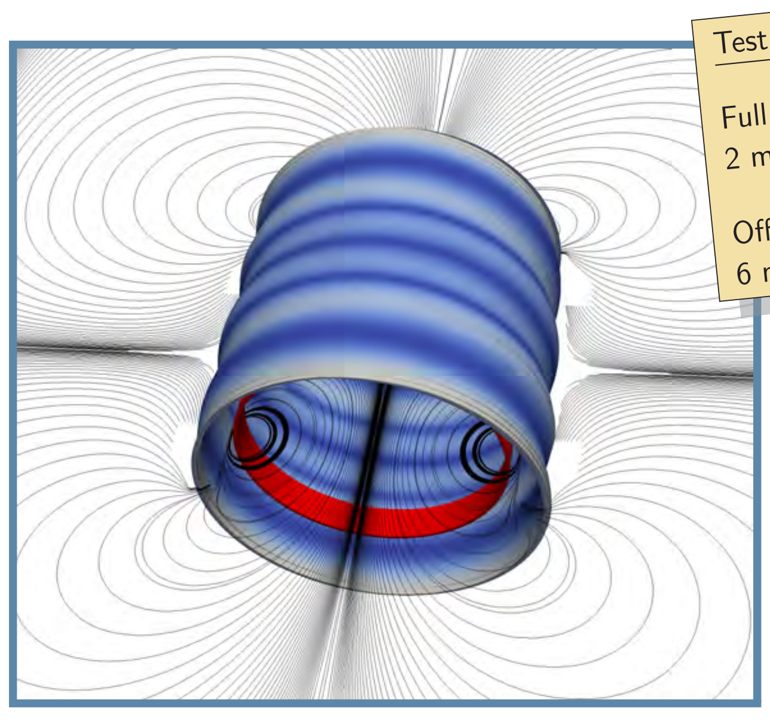
Full magnet

Full order:
45 min/f-sweep

PGD: x2
Offline: 90 min
Online: 15 s



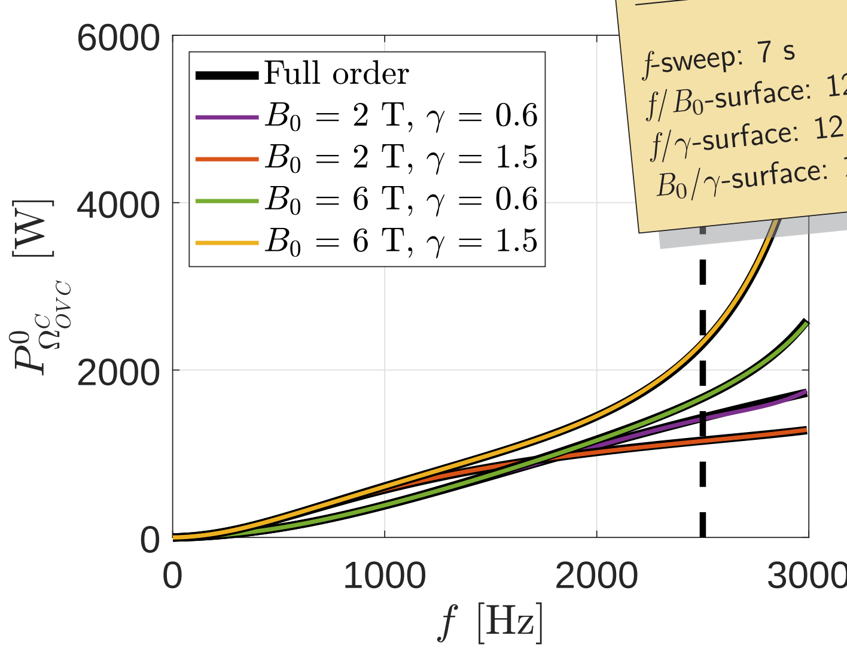
5D parametric test magnet problem $q^N(r, z, \omega, B_0, \gamma)$



Test magnet

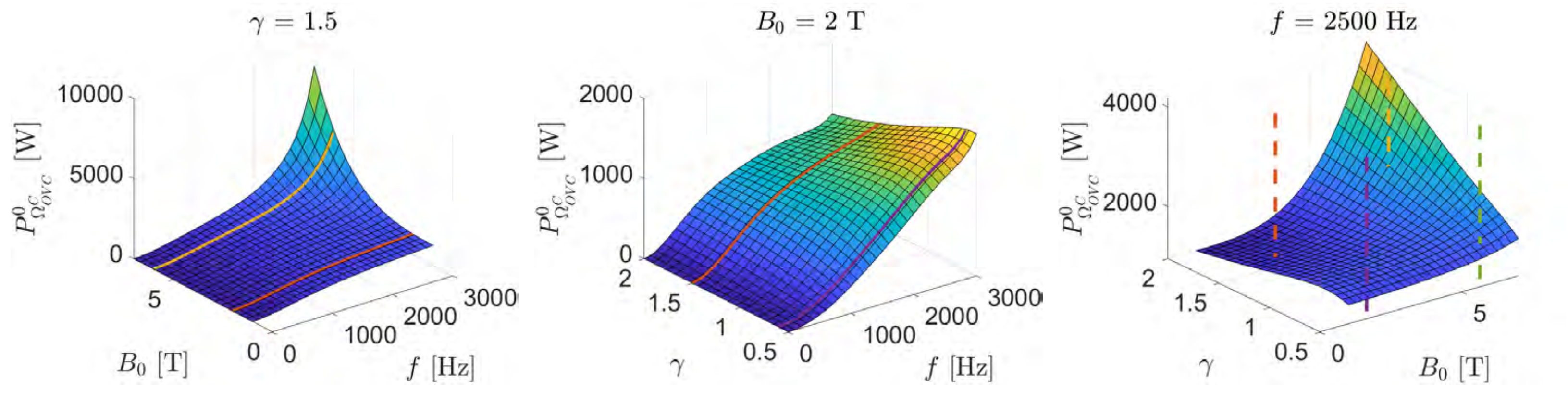
Full order:
2 min/f-sweep

Offline PGD: x3
6 min



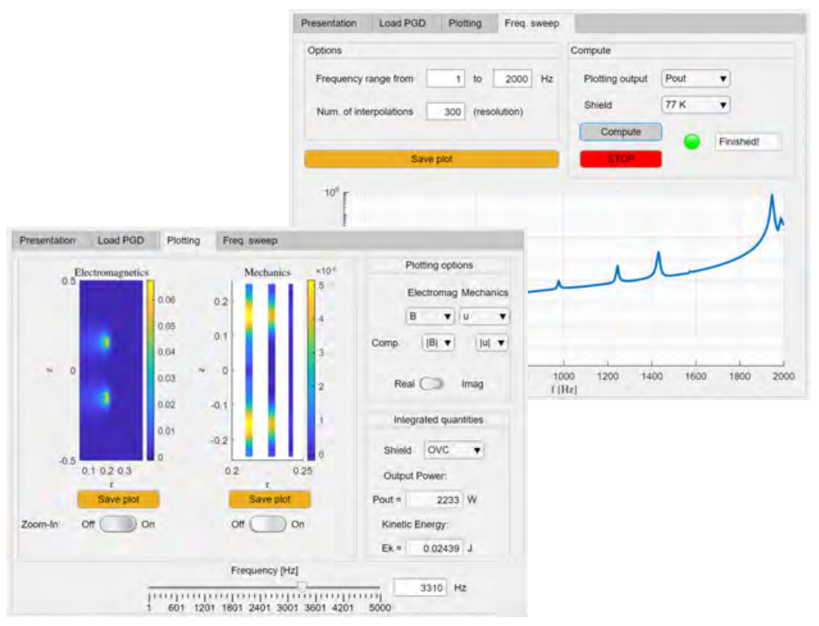
Online PGD

f-sweep: 7 s
f/B_0-surface: 12 s
f/γ-surface: 12 s
B_0/γ-surface: 12 s



6. Further work

- Further develop the **online PGD tool**
- **5D parametric PGD** in the **resonance region**
- **Geometrically parametrised PGD** (shield thickness)



7. References & Acknowledgements

1. Bagwell S, Ledger PD, Gil AJ, Mallett M, Kruip M. *A linearised hp-finite element framework for acousto-magneto-mechanical coupling in axisymmetric MRI scanners*. IJNME. 2017; 112(10):1323-52.
2. **Barroso G**, Gil AJ, Ledger PD, Mallet M, Huerta A. *A regularised-adaptive proper generalised decomposition for coupled magneto-mechanical problems with application to MRI scanners*. In preparation.

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`guillem.barroso@swansea.ac.uk`