High-fidelity real-time engineering outputs and sensitivities for aerodynamic shape design

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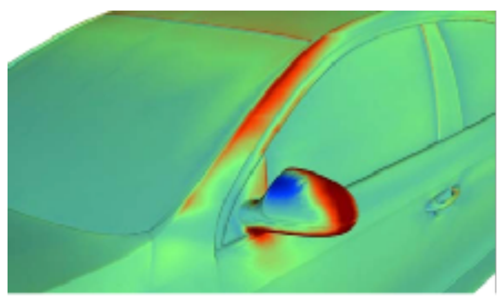
Motivations

- *Why high-order?*
  - high accuracy with complex physics and geometry
  - low dispersion and dissipation error

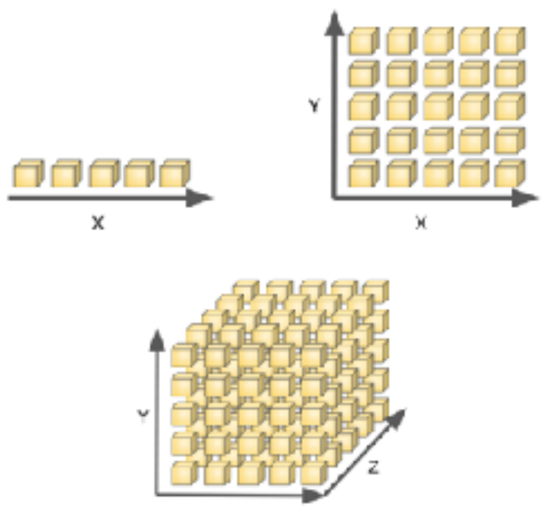
→ *high-order Hybridizable Discontinuous Galerkin* (***HDG***)



"Courtesy of Carsten Othmer (Volkswagen AG)"

- *Why model order reduction?*
  - necessity of testing different parametric settings
  - general vademecum with real time evaluation
  - circumvent the "curse of dimensionality"

→ *Proper Generalized Decomposition* (***PGD***)



"Curse of dimensionality"

Problem statement

- Considering the steady **Stokes** equation on Ω^μ for a 2nd order method

$$\begin{cases} -\nabla\cdot(\nu\nabla\mathbf{u}-p\mathbf{I})=\mathbf{0} & \text{in } \Omega^\mu,\\ \nabla\cdot\mathbf{u}=0 & \text{in } \Omega^\mu,\\ \mathbf{u}=\mathbf{u}_D & \text{on } \Gamma^\mu_D,\\ \mathbf{n}^\mu\cdot(\nu\nabla\mathbf{u}-p\mathbf{I})=\mathbf{0} & \text{on } \Gamma^\mu_N. \end{cases} \qquad \Omega(X,Y)\overset{\mathcal{M}_{\mu_1,\mu_2}}{\longmapsto}\Omega^\mu(x,y)$$

- By mapping the integrals that appear in a weak form on the reference domain:

$$\left(\nabla\mathbf{w},\nabla\mathbf{u}\right)_{\Omega_\mu}=\left(\mathbf{J}_\mu^{-1}\nabla_X\mathbf{w},\mathbf{J}_\mu^{-1}\nabla_X\mathbf{u}\,|\mathbf{J}_\mu|\right)_\Omega=\left(\mathbf{J}_\mu^{-1}\nabla_X\mathbf{w}\,\mathrm{adj}(\mathbf{J}_\mu)\nabla_X\mathbf{u}\right)_\Omega \ ✗$$

$$\left(\mathbf{w},\nabla\mathbf{u}\right)_{\Omega_\mu}=\left(\mathbf{w},\mathbf{J}_\mu^{-1}\nabla_X\mathbf{u}\,|\mathbf{J}_\mu|\right)_\Omega=\left(\mathbf{w},\mathrm{adj}(\mathbf{J}_\mu)\nabla_X\mathbf{u}\right)_\Omega \ ✓$$

$$\left(\mathbf{w},\mathbf{u}\right)_{\Omega_\mu}=\left(\mathbf{w},\mathbf{u}|\mathbf{J}_\mu|\right)_\Omega \ ✓$$

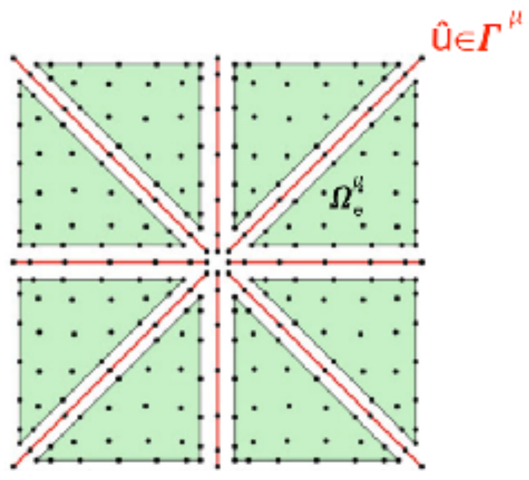
→ $\mathbf{J}_\mu^{-1}$ **requires HO-SVD, computationally very expensive**

We know the exact separated representations of $|\mathbf{J}_\mu|$, $\mathrm{adj}(\mathbf{J}_\mu)$ but not for $\mathbf{J}_\mu^{-1}$ [3,4].

HDG mixed formulation

- In a mixed method framework as HDG [2] instead:
  - express $\mathbf{u}$ and p in terms of $\hat{\mathbf{u}}$ and ρ

$$\begin{cases} \mathbf{L}=-\nabla\mathbf{u} & \text{in } \Omega^\mu_e,\\ \nabla\cdot(\nu\mathbf{L}+p\mathbf{I})=\mathbf{0} & \text{in } \Omega^\mu_e,\\ \nabla\cdot\mathbf{u}=0 & \text{in } \Omega^\mu_e,\\ \mathbf{u}=\mathbf{u}_D & \text{on } \Gamma^\mu_D,\\ \mathbf{u}=\hat{\mathbf{u}} & \text{on } \Gamma^\mu,\\ \langle p,1\rangle_{\partial\Omega^\mu_e}=\rho_e, \end{cases}$$



ndof DG vs ndof HDG

  - solve for $\hat{\mathbf{u}}$ and ρ

$$\begin{cases} [\![\mathbf{n}^\mu\cdot(\nu\mathbf{L}(\hat{\mathbf{u}})+p(\hat{\mathbf{u}})\mathbf{I})]\!]=\mathbf{0} & \text{on } \Gamma^\mu\\ \mathbf{n}^\mu\cdot(\nu\mathbf{L}(\hat{\mathbf{u}})+p(\hat{\mathbf{u}})\mathbf{I})=\mathbf{0} & \text{on } \Gamma^\mu_N. \end{cases}$$

- The mixed form enables an explicit separation of all the terms in the weak form

$$\left(\mathbf{w},\nabla\cdot\mathbf{L}\right)_{\Omega^\mu}=\left(\mathbf{w},\mathbf{J}_\mu^{-1}\nabla_X\cdot\mathbf{L}\,|\mathbf{J}_\mu|\right)_\Omega=\left(\mathbf{w},\mathrm{adj}(\mathbf{J})\nabla_X\cdot\mathbf{L}\right)_\Omega \ ✓ \ \textbf{separable}$$

References

1. Ammar A., Mokdad B., Chinesta F. and Keunings R., "A new family of solvers for some classes of multidimensional partial differential equations encountered in kinetic theory modelling of complex fluids."
2. Cockburn B., Gopalakrishnan J. and Lazarov R., "Unified hybridization of discontinuous Galerkin, mixed, and continuous Galerkin methods for second order elliptic problems."
3. Dìez P., Zlotnik S. and Huerta A., Generalized parametric solutions in Stokes flow.
4. Zlotnik S., Dìez P., Modesto D. and Huerta A., "Proper Generalized Decomposition of a geometrically parametrized heat problem with geophysical applications."
5. Avron J. E., O. Kenneth, and D. H. Oaknin. "Pushmepullyou: an efficient micro-swimmer."

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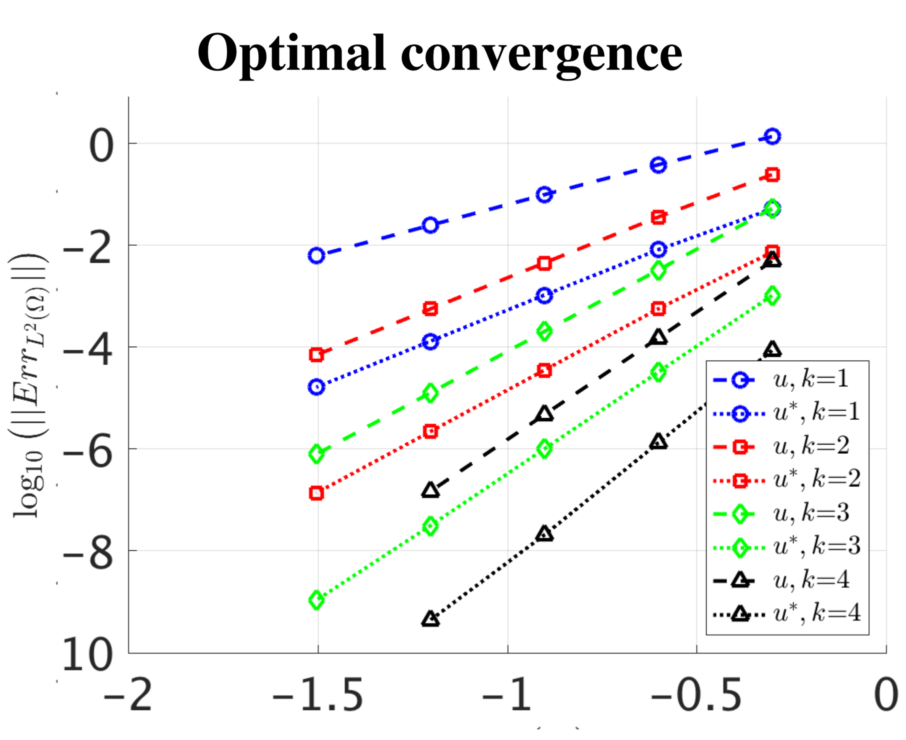
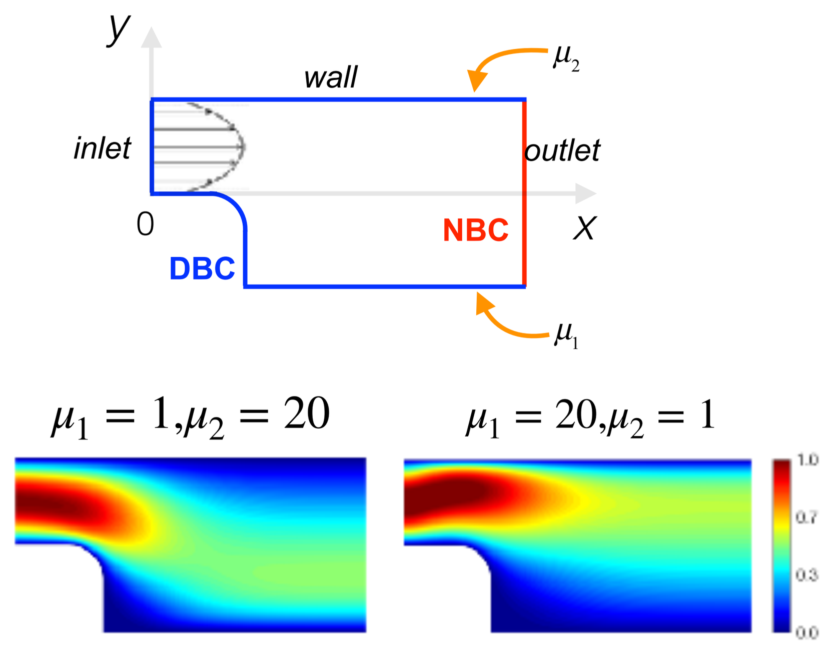
Results I year: parametric spatial dependent viscosity

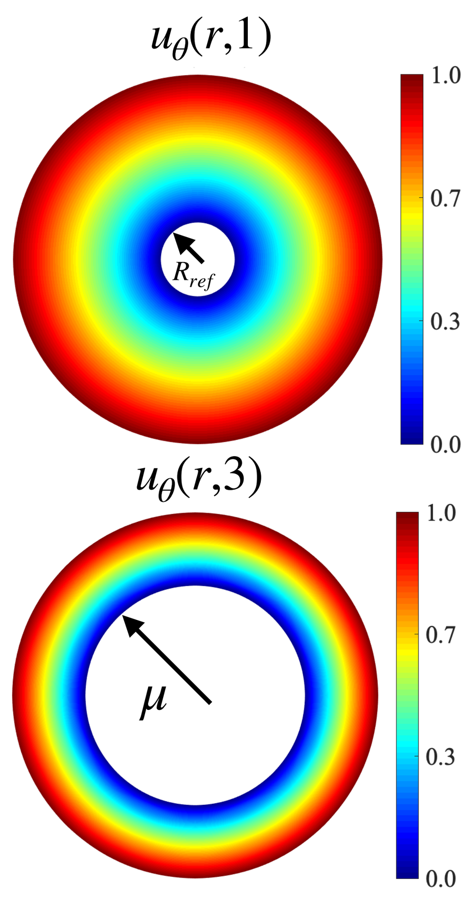
- Code validation for synthetic problem and application:

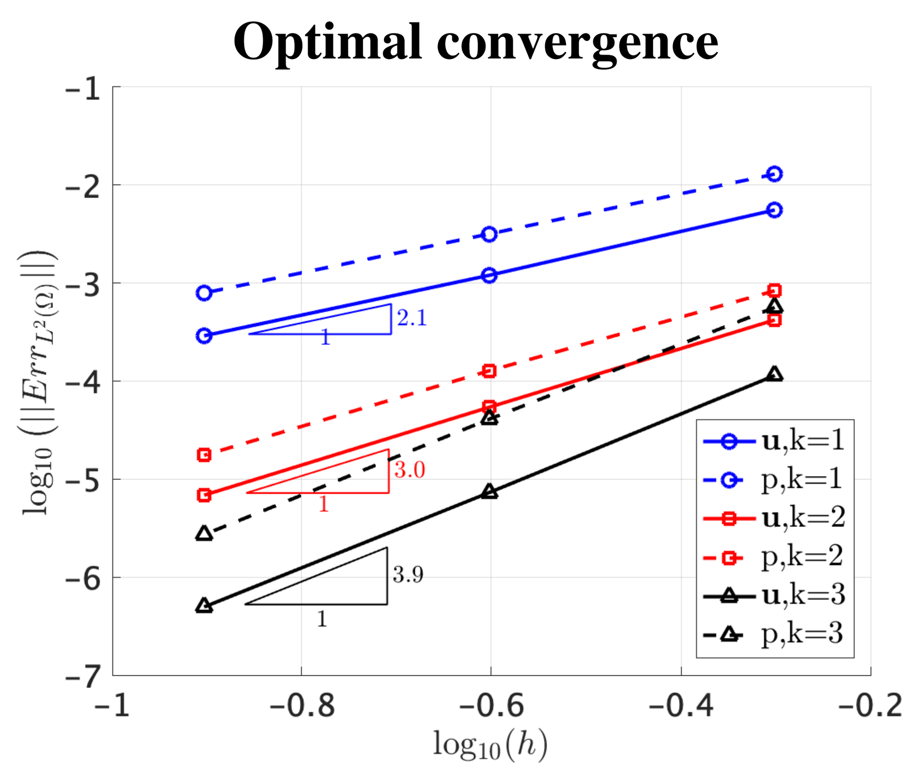




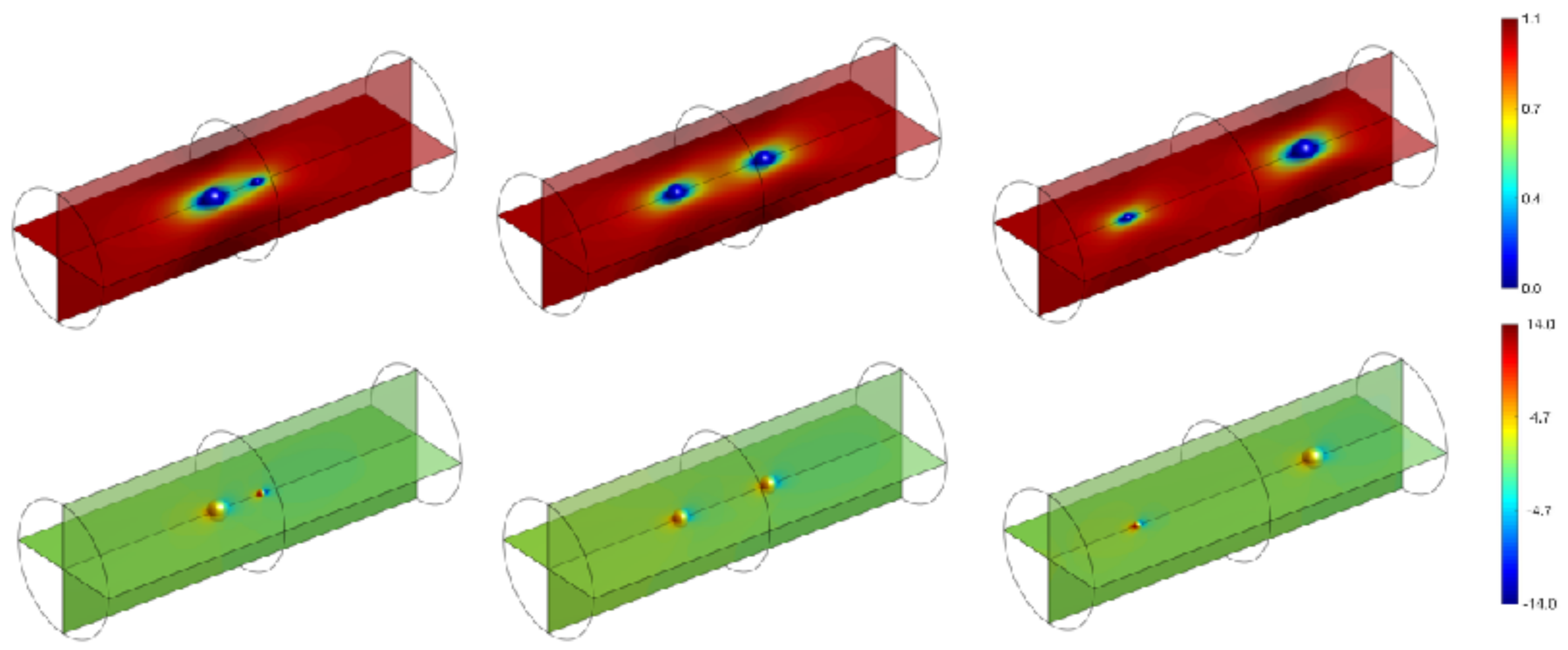
Results II year: 2D and Axi parameterised geometry

- Code validation for parametric Couette flow with $R_{in}=\mu$:



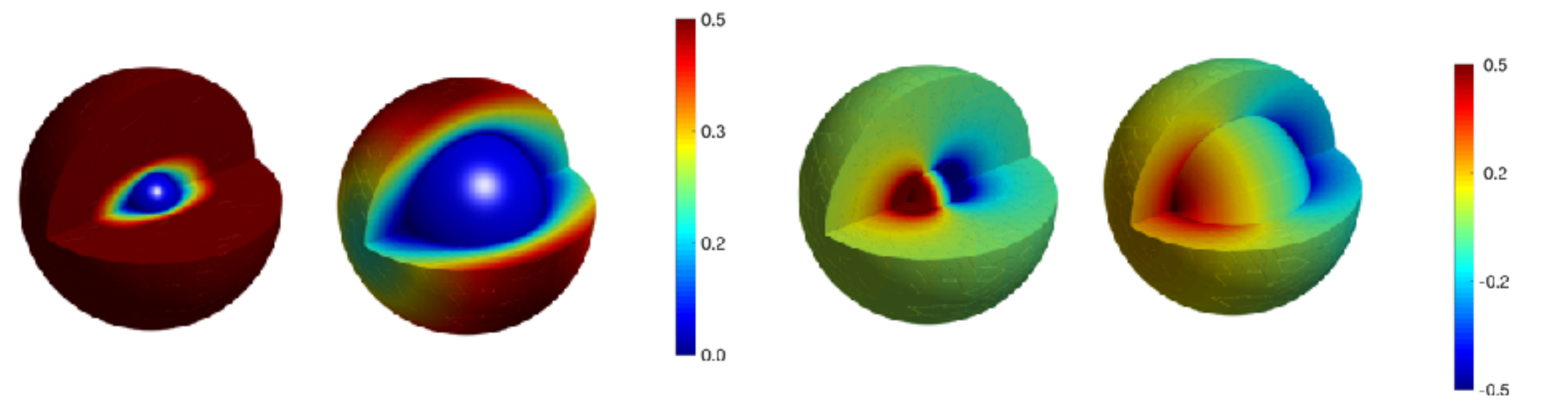


- Micro-swimmer HDG-PGD solutions for *k=3* for different configurations:



Results II year: 3D parameterised geometry

- Code validation for 3D problems with parameterised geometry: $R_{in}=\mu$



- Application to a corrugated channel with parameterised wave amplitude: