A hp-finite element software for the numerical simulation of 3D coupled magneto-mechanical problems with application to MRI scanners

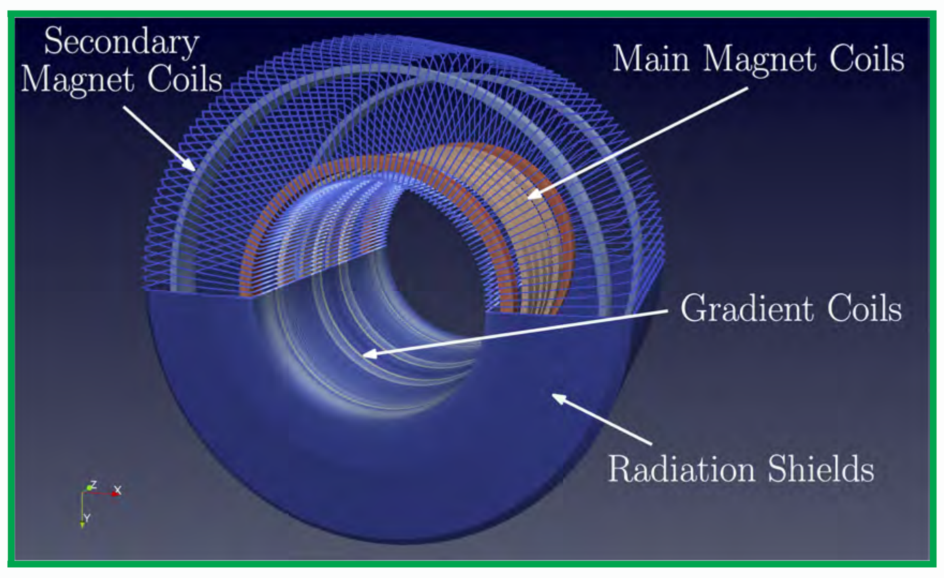
Marcos Seoane Chouciño[†,*,**]

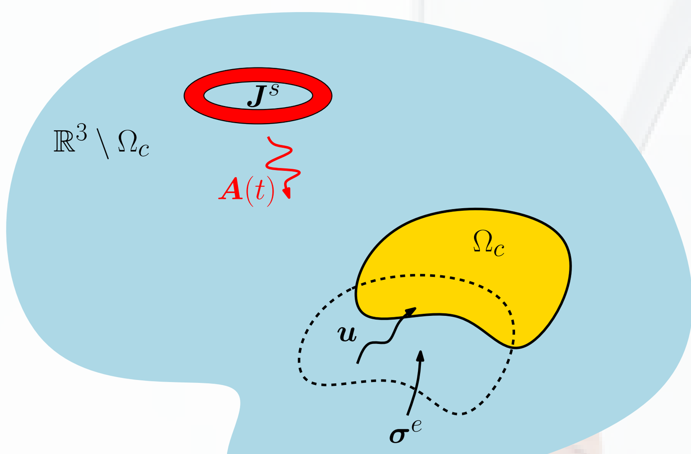
Supervisors: Paul D. Ledger[*], Antonio J. Gil[*], Mike J. Mallett[†], Sergio Zlotnik[**]

[*] Zienkiewicz Centre for Computational Engineering, Swansea University, UK   [†] Siemens Magnet Technology, UK   [**] Universitat Politècnica de Catalunya, Spain

Motivation

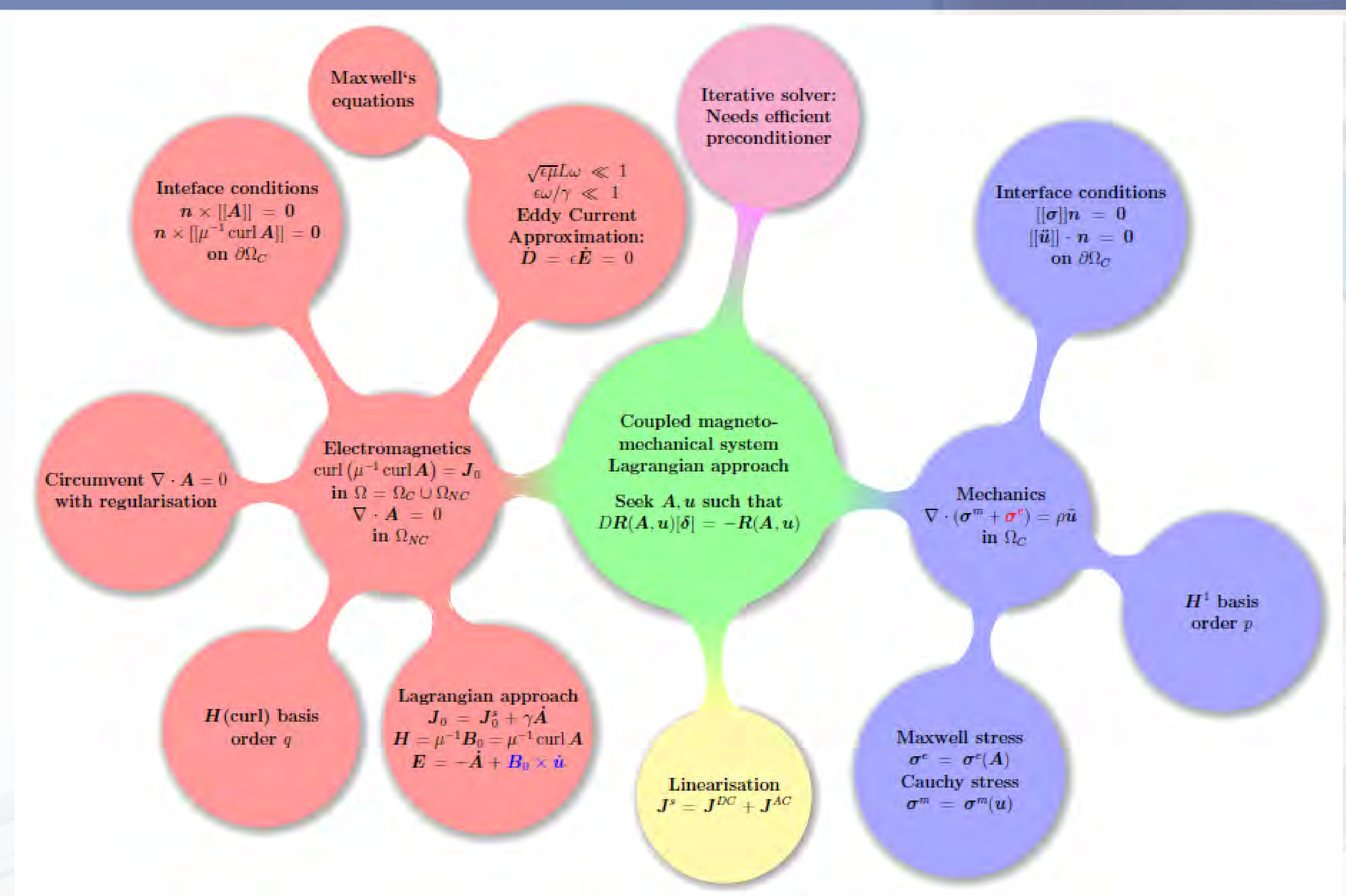
- To develop a software to **accurately** solve **3D coupled magneto-mechanical** problems with application to **MRI scanner design**
- **Industrial** Motivation
  - Transient magnetic fields induce **eddy currents** and **electromagnetic** stresses in conducting components
  - These eddy currents and magnetic stresses give rise to **vibrations**, **deformations** and **heat dissipation**
  - These effects are undesirable as they produce **image artefacts**, **patient discomfort** and **helium boil-off**





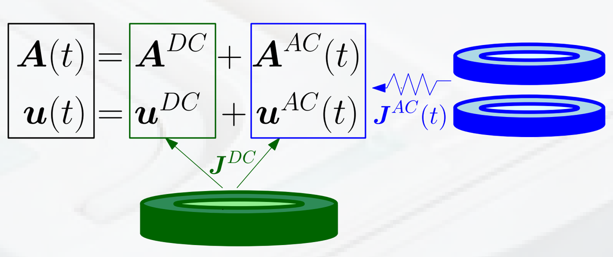
- **Computational** Motivation
  - Develop numerical tool to solve fully coupled 3D magneto-mechanical problems in the frequency domain.
  - **Accurate** simulations using *hp*-FEM
- **MRI Scanner** components
  - **Main static** magnetic coils
  - **Gradient** coils (X, Y & Z)
  - Radiation **shields**

Coupled System



Linearisation and Solver

- Linearisation

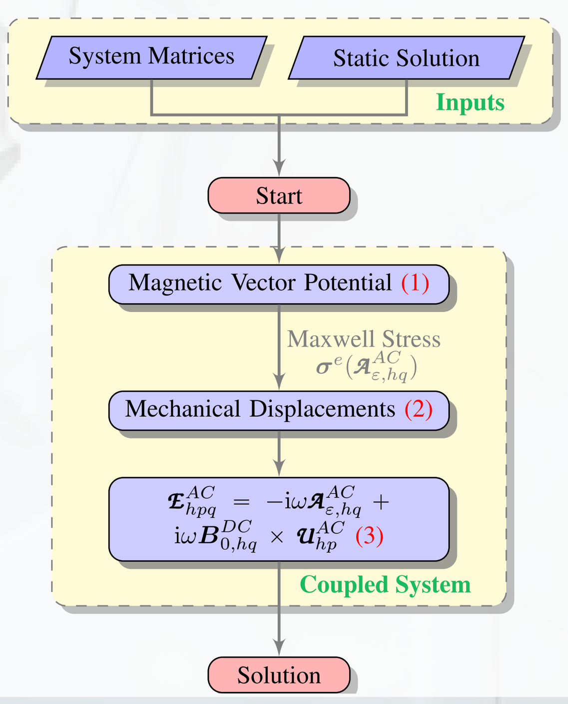


$$A(t) = A^{DC} + A^{AC}(t) = A^{DC} + \mathrm{Re}(\mathcal{A}^{AC} e^{i\omega t})$$
$$u(t) = u^{DC} + u^{AC}(t) = u^{DC} + \mathrm{Re}(\mathcal{U}^{AC} e^{i\omega t})$$

- Physical electric and magnetic fields

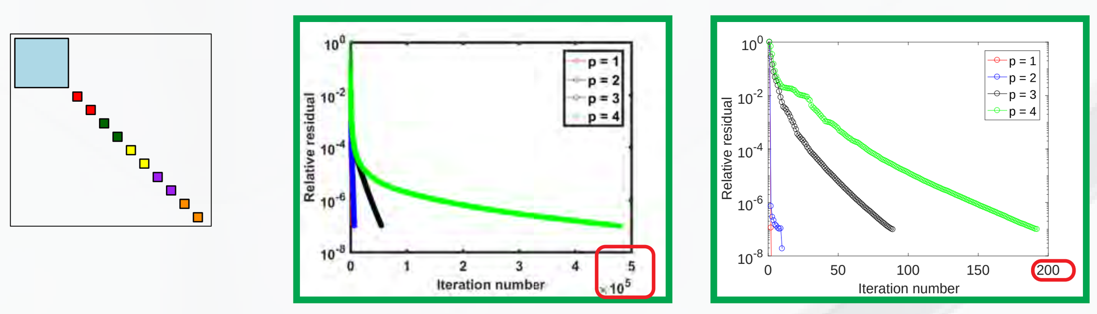
$$E(t) = \mathrm{Re}((-i\omega \mathcal{A}^{AC} + i\omega B_0^{DC} \times \mathcal{U}^{AC}) e^{i\omega t})$$
$$H(t) = \mu^{-1}(\mathrm{curl}\, A^{DC} + \mathrm{Re}(e^{i\omega t}\, \mathrm{curl}\, \mathcal{A}^{AC}))$$

- Solver scheme



(1) EM system is ill conditioned due to regularisation → Preconditioned GMRES iterative solver
(2) Mechanical system indefinite and smaller than EM system → Direct Solver
(3) Electric field coupled with displacement

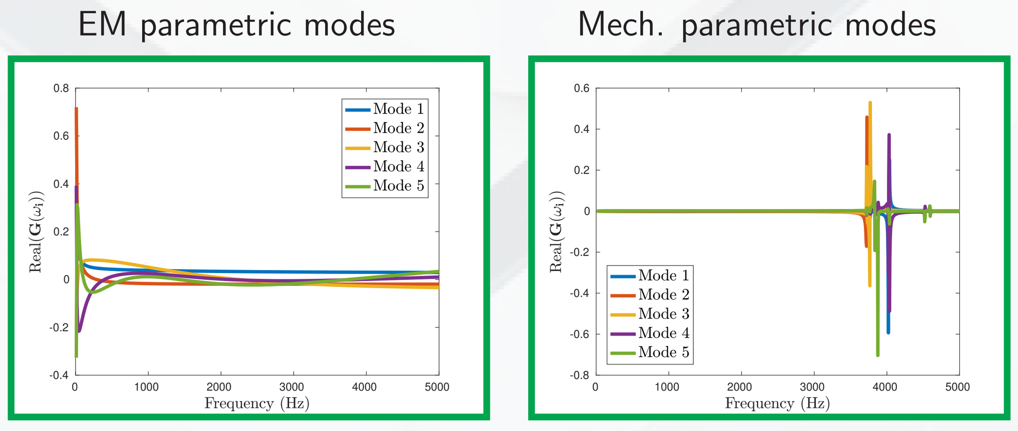
No preconditioning | Preconditioned



Reduced Order Modelling: Proper Orthogonal Decomposition

- Express the solution in a separate format: $q(x,\omega) \approx q_M = \sum_{i=1}^{M} f_i(x) g_i(\omega)$
- Electromagnetic solution is much smoother than mechanical:

EM parametric modes | Mech. parametric modes



- Combined reduced order-full order approach: POD for electromagnetic problem and full order for mechanical problem.

Outputs of Interest: Dissipated Power and Kinetic Energy

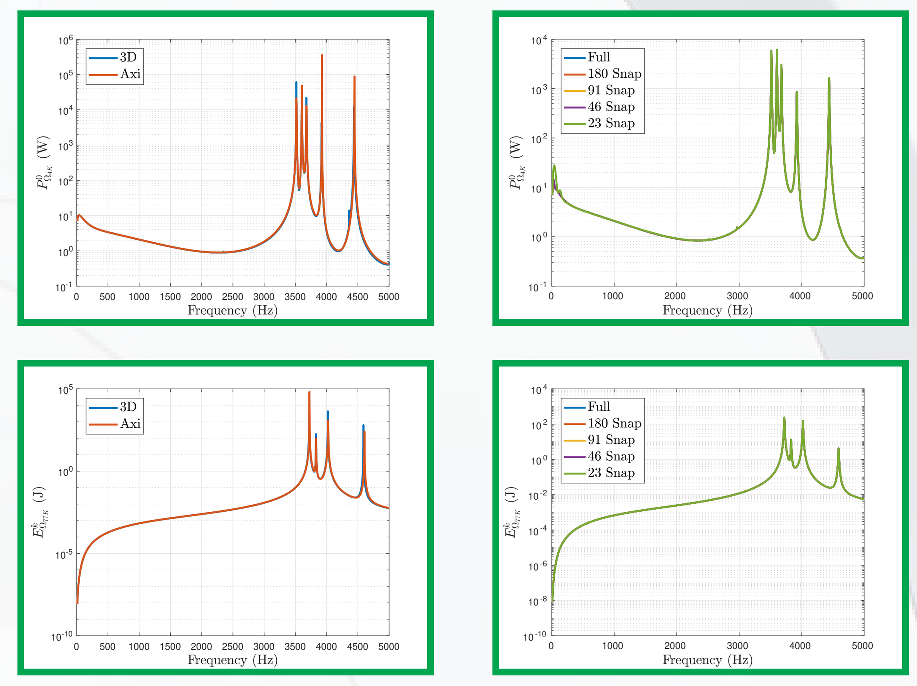
- **Dissipated Power**

$$P^0_{\Omega_C} = \frac{1}{2} \int_{\Omega_C} \gamma \left| \mathcal{E}(\omega, \mathcal{A}^{AC}, \mathcal{U}^{AC}) \right|^2 d\Omega$$

- **Kinetic Energy**

$$E^k_{\Omega_C} = \frac{1}{2} \int_{\Omega_C} \rho\omega^2 \left| \mathcal{U}^{AC} \right|^2 d\Omega.$$

Validation | Reduced vs full

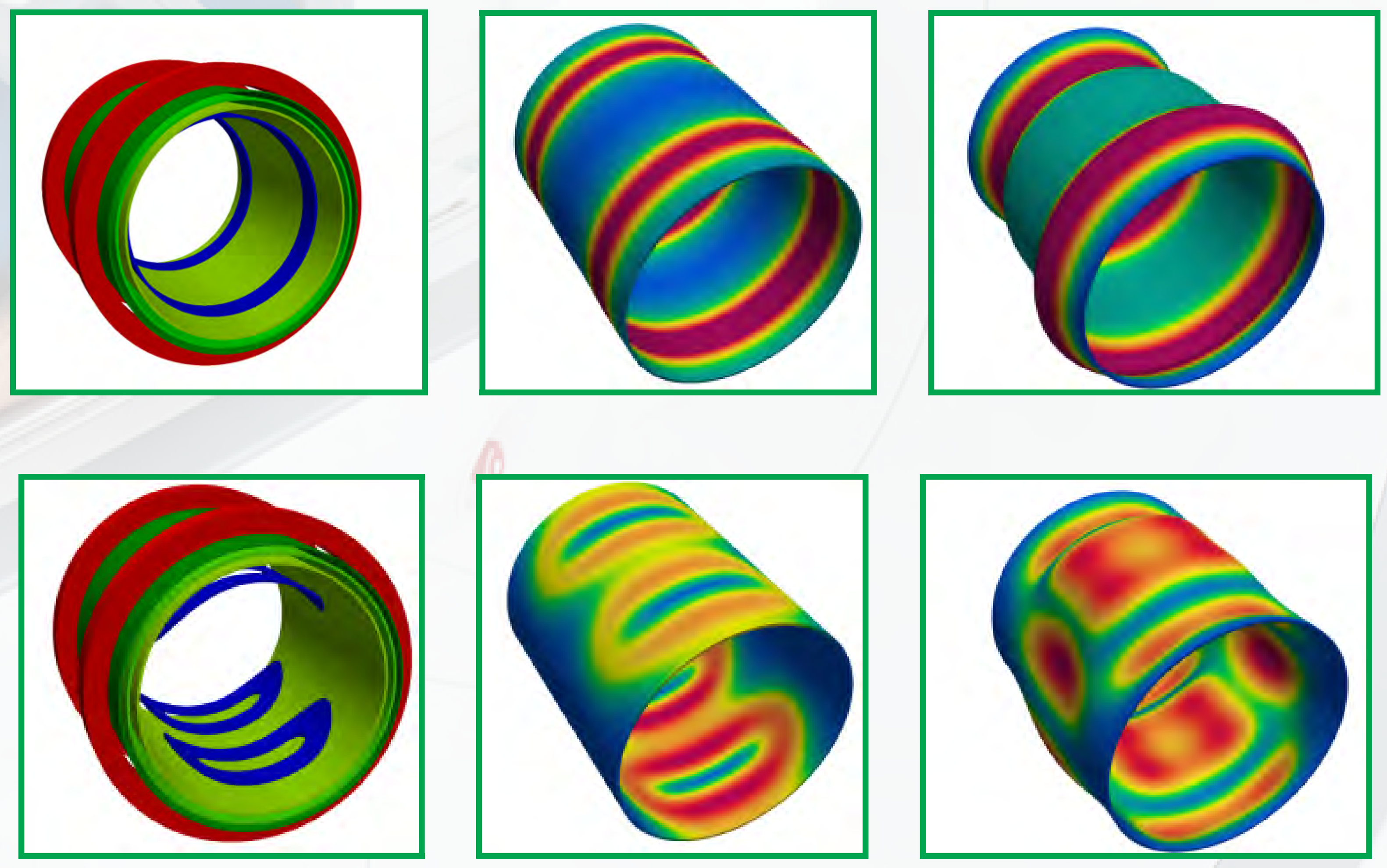


- Full order magnetic vector potential computed only at a **reduced number of snapshots**
- Full order solution for displacements computed at **700 frequencies**

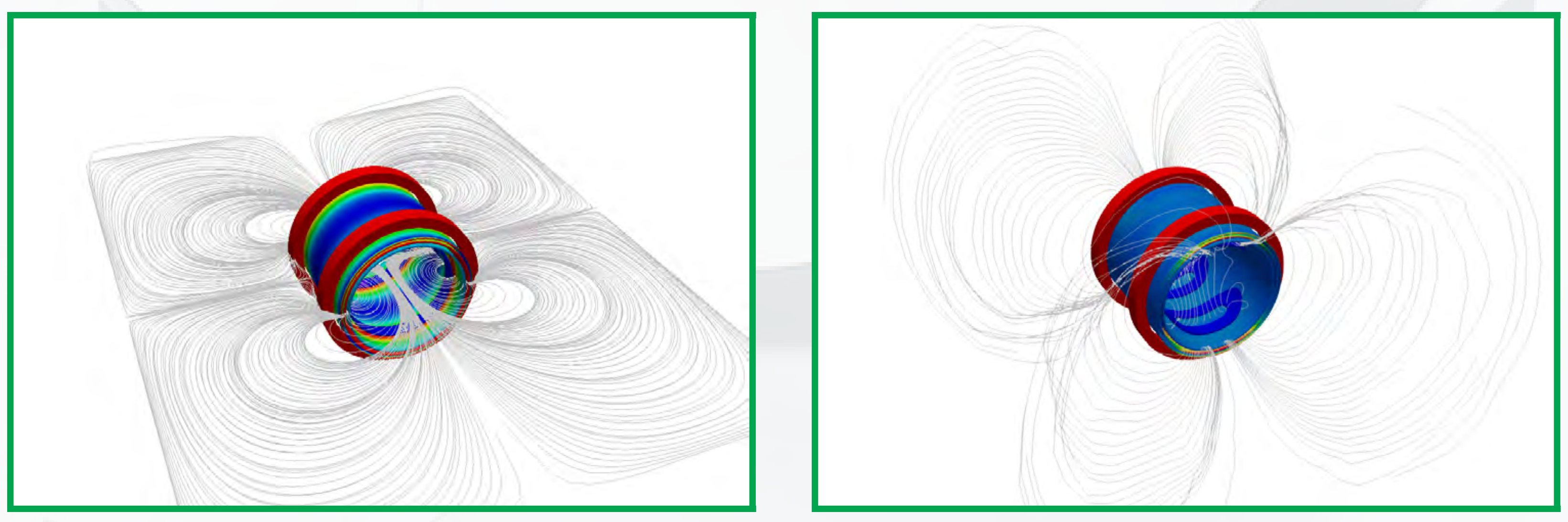
Model	Full	180 Snapshots	91 Snapshots	46 Snapshots	23 Snapshots
Solver Time (h)	6.7	2.9	2.4	2.1	2
Speed-up (%)	0	57	64	69	70

Longitudinal vs Transversal Gradient Coils

Geometry | Eddy currents | Displacements



- **Magnetic field streamlines and eddy currents on the three shields**



Conclusions and Future Work

- **Conclusions**
  - **3D** *hp*-finite element formulation for coupled **magneto-mechanical** problems developed and validated
  - **Efficient preconditioner** for electromagnetic system
  - *hp*-finite elements lead to accurate solutions
  - **Lagrangian** approach is **computationally efficient**
  - Combined **reduced order-full order** approach leads to a big **reduction in computing time**
- **Ongoing and Future Work**
  - Solve more **complex MRI configurations**
  - Study the application of **reduced basis** considering **more than one parameter**
  - Application of **algebraic PGD** to coupled problem

References

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LinkedIn profile: Marcos Seoane Chouciño

Email: marcos.seoane_choucino@siemens-healthineers.com