Exact bounds for functional outputs of parabolic problems: application to the transient convection-diffusion-reaction equation

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ABSTRACT

One of the new paradigms in computational adaptivity is to obtain and control functional outputs of the exact solutions of boundary value and/or evolution problems [1,2,3]. Although the exact solution of the problem is obviously unknown, bounds of the functional outputs are computed from the approximate finite element solutions using error assessment techniques. In particular, for transient convection-diffusion-reaction simulations, the strategy presented in [4] allows obtaining asymptotic bounds for the functional outputs of a reference solution (asymptotic is referred to the reference solution, that is not exact but as if it was computed with a much finer truth mesh) and assuming that the error introduced by the time marching scheme is negligible.

The methodology presented here produces bounds for linear functional outputs of the solution of the transient convection-diffusion-reaction equation. The bounds are not asymptotic, that is with respect to the solution computed with a much finer mesh, but exact. In a first phase, the assumption of neglecting the error introduced in the time integration is kept. Secondly, this assumption is removed and, following a more sophisticated approach, the output is computed accounting also for the error arising from the time discretization.

References

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