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EXACT ERROR BOUNDS FOR LINEAR OUTPUTS OF THE CONVECTION-DIFFUSION-REACTION EQUATION USING FLUX-FREE ERROR ESTIMATORS

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ABSTRACT

The Flux-free approach is a promising alternative to standard implicit residual time error estimators that require the equilibration of hybrid fluxes. The idea is to solve local error problems in patches of elements surrounding one node (also known as stars) instead of in single elements [1]. The resulting local problems are flux-free, that is the boundary conditions are natural and hence their implementation is straightforward. This allows precluding the computation and the equilibration of fluxes along the element edges (tractions in a mechanical context). The domain decomposition is performed using a partition of the unity strategy. The resulting estimates are much simpler from the implementation viewpoint, especially in the 3D cases, and provide upper bounds of the energy norm of the error (as well as the standard implicit residual estimators with equilibration of hybrid fluxes). In the past, the local flux-free problems have been solved using a finite element mesh inside each local subdomain. Consequently, the resulting estimates were asymptotic upper bounds (with respect to some reference solution) rather than exact upper bounds (with respect to the exact solution).

Some effort has been devoted to recover exact upper bound using the equilibrated hybrid fluxes approach. The idea is to solve the local problem using a dual formulation and to minimize the complementary energy [2]. Here, the same idea is employed to obtain exact upper bounds using the flux-free approach. The resulting estimates have similar features as their asymptotic version, while providing a guaranteed upper bound.

In the context of convection-diffusion-reaction problems, reference [3] provides a methodology to derive guaranteed bounds using the equilibrated hybrid fluxes approach. This technique uses as input of the error estimation procedure the Galerkin (not stabilized) finite element approximation of the problem. It is well known however, that Galerkin approximations to convection dominated problems are often corrupted by spurious node-to-node oscillations and in practical applications stabilization techniques are needed to properly approximate the solution. Here, following ideas given in [4], the methodology

introduced in [3] is modified such that it accepts as input the stabilized solutions. Although the rationale of the proposed modifications is different to the original approach, it is worth noting that the implementation is similar and the same codes can be used for both purposes.

Finally, in the present work, a new methodology is introduced providing guaranteed bounds of the stabilized solutions using the flux-free domain decomposition.

The behavior of the bounds obtained using both the equilibrated hybrid fluxes approach and the flux-free approach is analyzed for the quasi-2D convection-reaction-diffusion problem introduced in [3]. In particular, the effect of including the convective term is studied for different values of the velocity $\boldsymbol{\alpha}$. For all the values of $\boldsymbol{\alpha}$, the rate of convergence of the bound gap is the expected. However, the bound gap is larger as $\boldsymbol{\alpha}$ increases, as can bee seen in figure 1. This increment does not correspond to the actual error increment and therefore the efficiency of the bounds is deteriorated if the convection parameter is large. This behavior is alleviated when the stabilized approaches are used.

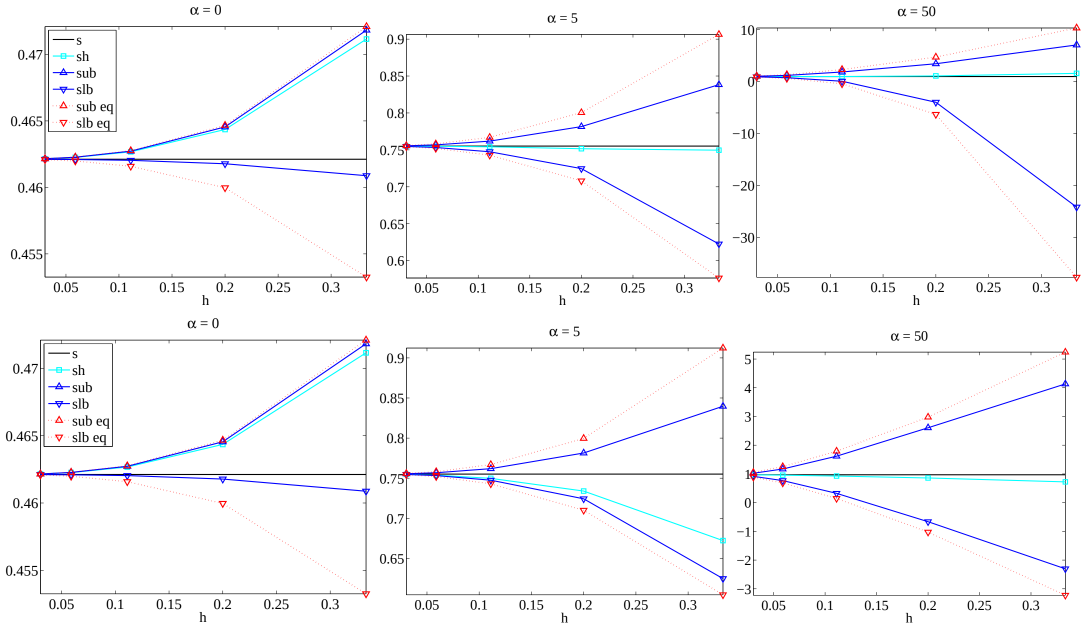

Figure 1: Non-stabilized (top) and stabilized (bottom) bounds in a series of uniformly h-refined meshes for a convection parameter $\boldsymbol{\alpha} = (\alpha, 0)$, $\alpha = 0, 5$ and 50 and for a reaction and diffusion parameters $\sigma = 1$ and $\nu = 1$. s and *sh* stand for the exact and finite element approximation of the output and *slb*, *sub*, *slb eq* and *sub eq* are the lower and upper bounds for s computed using the flux-free and the hybrid-flux techniques respectively.

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