**Flux-free error estimators and adaptivity for finite elements and mesh-free methods**

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Assessment of functional outputs of the solution (goal-oriented error estimation) in *computational mechanics* problems is a real need in standard engineering practice. In particular, end-users of finite elements, finite differences or mesh-free codes are interested in obtaining bounds for quantities of engineering interest. Techniques providing these bounds require using error estimators in the energy norm of the solution. Bounds for quantities of interest (functional outputs) are recovered combining upper and lower bounds of the energy error for both the original problem (primal) and a dual problem (associated with the selected functional output).

The need of obtaining reliable upper and lower bounds of the error for quantities of interest has motivated the use of residual error estimators, which are currently the only type of estimators ensuring bounds for the error. Classical residual type estimators, which provide upper bounds of the error, require flux-equilibration procedures (*hybrid-flux* techniques) to properly set boundary conditions for local problems. Flux-equilibration requires a domain decomposition, which is natural in finite elements but not in mesh-free methods. And, moreover, it is performed by a complex algorithm, strongly dependent on the finite element type and requiring a data structure that is not natural in a standard finite element code. Thus highly embedded in the finite element domain decomposition.

The main advantages of flux-free estimators, see for instance [1], are the simplicity in the implementation and that the extension to mesh-free methods is possible. This is a concept that also exists in mesh-free methods and thus the extension is possible. Moreover, boundary conditions of the local problems are trivial and the usual data structure of a code is directly employed.

To the authors knowledge implicit residual based estimators have not been proposed for mesh-free methods [2]. However, these residual based approaches are now standard in finite elements because they are more mathematically sound, more precise and allow to compute upper and lower bounds for energy norms as well as functional outputs.

Here a unified discussion is presented on implicit residual-type flux-free error estimator proposed for finite elements and mesh-free. Moreover is it shown that similar efficiency as standard hybrid-flux estimators is achieved. References

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